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SAMPLE PAPERS

CBSE EXAM 2025

20 Sets

Class : 12th

Sub : Maths

How to see answers or marking scheme ?

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Sample Paper 01

Class - 12th Exam - 2024 - 25

Mathematics (Code-041)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided some questions.
9. Use of calculators is not allowed.

Section - A

Section A consists of 20 questions of 1 mark each.

1. If $f(x) = \log_e(\log_e x)$, then $f'(e)$ is equal to

(a) e^{-1} (b) e

(c) 1 (d) 0

2. The degree of the differential equation $x = 1 + \left(\frac{dy}{dx}\right) + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + \ldots$, is

(a) 3 (b) 2

(c) 1 (d) not defined

3. The symmetric part of the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ 6 & 8 & 2 \\ 2 & -2 & 7 \end{bmatrix}$ is equal to

(a) $\begin{bmatrix} 0 & -2 & -1 \\ -2 & 0 & -2 \\ -1 & -2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 4 & 3 \\ 2 & 8 & 0 \\ 3 & 0 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & 2 \\ -1 & 2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 4 & 3 \\ 4 & 8 & 0 \\ 3 & 0 & 7 \end{bmatrix}$

4. The value of $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$ is

(a) $\frac{x^2}{2} + \log|x| - 2x + C$ (b) $\frac{x^2}{2} + \log|x| + 2x + C$

(c) $\frac{x^2}{2} - \log|x| - 2x + C$ (d) None of these

5. The solution of $\frac{dy}{dx} = \frac{ax+g}{by+f}$ represents a circle, when

(a) $a = b$ (b) $a = -b$

(c) $a = -2b$ (d) $a = 2b$

6. If $y = \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ is

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) 1 (d) -1

7. The area of enclosed by $y = 3x - 5$, $y = 0$, $x = 3$ and $x = 5$ is

(a) 12 sq units (b) 13 sq units

(c) $13\frac{1}{2}$ sq units (d) 14 sq units

8. The general solution of the differential equation $\frac{dy}{dx} = e^y(e^x + e^{-x} + 2x)$ is

(a) $e^{-y} = e^x - e^{-x} + x^2 + C$ (b) $e^{-y} = e^{-x} - e^x - x^2 + C$

(c) $e^{-y} = -e^{-x} - e^x - x^2 + C$ (d) $e^y = e^{-x} + e^x + x^2 + C$

9. If $\lambda(3\hat{i} + 2\hat{j} - 6\hat{k})$ is a unit vector, then the value of λ is

(a) $\pm\frac{1}{7}$ (b) ± 7

(c) $\pm\sqrt{43}$ (d) $\pm\frac{1}{\sqrt{43}}$

10. If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b}$ is a vector such that $\vec{a} \cdot \vec{b} = |\vec{b}|^2$ and $|\vec{a} - \vec{b}| = \sqrt{7}$, then $|\vec{b}|$ is equal to

(a) $\sqrt{7}$ (b) $\sqrt{3}$

(c) 7 (d) 3

11. The area of the region bounded by the lines $y = mx$, $x = 1$, $x = 2$ and X-axis is 6 sq units, then m is equal to

(a) 3 (b) 1

(c) 2 (d) 4

12. The direction cosines of the line joining the points $(4,3,-5)$ and $(-2,1,-8)$ are

(a) $\left(\frac{6}{7},\frac{2}{7},\frac{3}{7}\right)$ (b) $\left(\frac{2}{7},\frac{3}{7},\frac{-6}{7}\right)$

(c) $\left(\frac{6}{7},\frac{3}{7},\frac{2}{7}\right)$ (d) None of these

13. The least, value of the function $f(x) = ax + b/x$, $a > 0$, $b > 0$, $x > 0$ is

(a) $\sqrt{ab}$ (b) $2\sqrt{\frac{a}{b}}$

(c) $2\sqrt{\frac{b}{a}}$ (d) $2\sqrt{ab}$

14. If the lines $\frac{1-x}{3} = \frac{y-2}{2\alpha} = \frac{z-3}{2}$ and $\frac{x-1}{3\alpha} = y-1 = \frac{6-z}{5}$ are perpendicular, then the value of α is

(a) $\frac{-10}{7}$

(b) $\frac{10}{7}$

(c) $\frac{-10}{11}$

(d) $\frac{10}{11}$

15. A bag A contains 4 green and 3 red balls and bog B contains 4 red and 3 green balls. One bag is taken at random and a ball is drawn and noted to be green. The probability that it comes from bag B is

(a) $\frac{2}{7}$

(b) $\frac{2}{3}$

(c) $\frac{3}{7}$

(d) $\frac{1}{3}$

16. Find the area of a curve $xy = 4$, bounded by the lines $x = 1$ and $x = 3$ and X-axis.

(a) $\log 12$

(b) $\log 64$

(c) $\log 81$

(d) $\log 27$

17. If $P(A) = \frac{4}{5}$, and $Q(A \cap B) = \frac{7}{10}$, then $P\left(\frac{B}{A}\right)$ is equal to

(a) $\frac{1}{10}$

(b) $\frac{1}{8}$

(c) $\frac{7}{8}$

(d) $\frac{17}{20}$

18. The point on the curve $x^2 = 2y$ which is nearest to the point $(0,5)$ is

(a) $(2\sqrt{2},4)$

(b) $(2\sqrt{2},0)$

(c) $(0,0)$

(d) $(2,2)$

19. **Assertion :** $\int \frac{dx}{e^x + e^{-x} + 2} = \frac{1}{e^x + 1} + C$

Reason : $\int \frac{d\{f(x)\}}{\{f(x)\}^2} = -\frac{1}{f(x)} + C$

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

20. **Assertion :** $\int \frac{xe^x}{(x+1)^2} dx = \frac{e^x}{x+1} + C$

Reason : $\int e^x \{f(x) + f'(x)\}\, dx = e^x f(x) + C$

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$.

22. Find the general solution of differential equation $y = e^{2x}(a + bx)$

23. Differentiate $\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$ with respect to x.

OR

If $y = \sin^{-1}(6x\sqrt{1-9x^2})$, $-\frac{1}{3\sqrt{2}} < x < \frac{1}{3\sqrt{2}}$, then find $\frac{dy}{dx}$.

24. Suppose a girls throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once gets notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4 ,5 or 6 with the die?

25. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Find the values of a and b such that the function defined as follows is continuous.

$$f(x) = \begin{cases} x+2, & x \leq 2 \\ ax+b, & 2 < x < 5 \\ 3x-2, & x \geq 5 \end{cases}$$

27. Let R be a relation defined on the set of natural numbers N as follow :

$$R = \{(x, y): x \in N, y \in N \text{ and } 2x + y = 24\}$$

Find the domain and range of the relation R. Also, find if R is an equivalence relation or not.

28. Find a vector of magnitude 5 units and parallel to the resultant of $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

29. Find $\int \frac{2x}{(x^2+1)(x^2+2)^2} dx$.

OR

Integrate w.r.t. x, $\frac{x^2 - 3x + 1}{\sqrt{1-x^2}}$.

30. Solve the following differential equation $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$

31. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, then find $(A')^{-1}$.

OR

Find the adjoint of the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that $A(adj\ A) = |A| I_3$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Find both the maximum value and minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0,3]$.

OR

At what points in the interval $[0, 2\pi]$, does the function $\sin 2x$ atain its maximum value?

33. Maximize $Z = -x + 2y$, Subject to the constraints : $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$

OR

Minimise $Z = x + 2y$ subject to $2x + y \geq 3$, $x + 2y \geq 6$, x, $y \geq 0$. Show that the minimum of Z occurs at more than two points.

34. Find the value of λ, so that the lines $\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not.

OR

Find the shortest distance between the lines

$$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k}) \text{ and } \vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k}).$$

35. Find the value of the following : $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

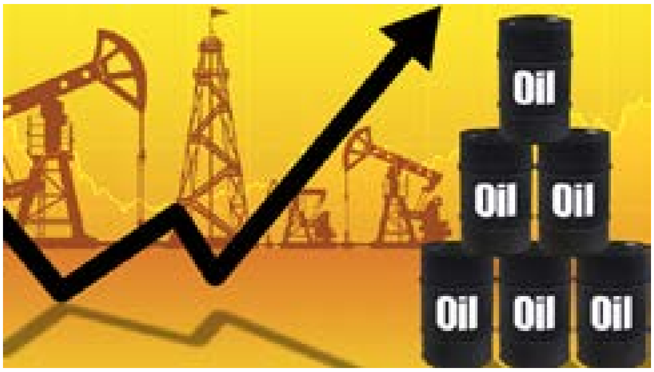
OR

Find the value of the following $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

Section - E

Case study based questions are compulsory.

36. Commodity prices are primarily determined by the forces of supply and demand in the market. For example, if the supply of oil increases, the price of one barrel decreases. Conversely, if demand for oil increases (which often happens during the summer), the price rises. Gasoline and natural gas fall into the energy commodities category.



The price p (dollars) of each unit of a particular commodity is estimated to be changing at the rate

$$\frac{dp}{dx} = \frac{-135x}{\sqrt{9+x^2}}$$

where x (hundred) units is the consumer demand (the number of units purchased at that price). Suppose 400 units $(x = 4)$ are demanded when the price is \$30 per unit.

(i) Find the demand function $p(x)$.

(ii) At what price will 300 units be demanded? At what price will no units be demanded?

(iii) How many units are demanded at a price of \$20 per unit?

37. Quality assurance (QA) testing is the process of ensuring that manufactured product is of the highest possible quality for customers. QA is simply the techniques used to prevent issues with product and to ensure great user experience for customers.

A manufactured component has its quality graded on its performance, appearance, and cost. Each of these three characteristics is graded as either pass or fail. There is a probability of 0.40 that a component passes on both appearance and cost. There is a probability of 0.35 that a component passes on both performance and appearance. There is a probability of 0.31 that a component passes on all three characteristics. There is a probability of 0.64 that a component passes on performance. There is a probability of 0.19 that a component fails on all three characteristics. There is a probability of 0.06 that a component passes on appearance but fails on both performance and cost.

(i) What is the probability that a component passes on cost but fails on both performance and appearance?

(ii) If a component passes on both appearance and cost, what is the probability that it passes on all three characteristics?

(iii) If a component passes on both performance and appearance, what is the probability that it passes on all three characteristics?

38. Sun Pharmaceutical Industries Limited is an Indian multinational pharmaceutical company headquartered in Mumbai, Maharashtra, that manufactures and sells pharmaceutical formulations and active pharmaceutical ingredients in more than 100 countries across the globe.

Sun Pharmaceutical produces three final chemical products P_1, P_2 and P_3 requiring mixup of three raw material chemicals M_1, M_2 and M_3. The per unit requirement of each product for each material (in litres) is as follows:

$$\begin{array}{cc} & \begin{array}{ccc} M_1 & M_2 & M_3 \end{array} \\ A = \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} & \begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 5 \\ 2 & 4 & 2 \end{bmatrix} \end{array}$$

(i) Find the total requirement of each material if the firm produces 100 litres of each product,

(ii) Find the per unit cost of production of each product if the per unit of materials M_1, M_2 and M_3 are ₹ 5, ₹ 10 and ₹ 5 respectively, and

(iii) Find the total cost of production if the firm produces 200 litres of each product.

□□□□□□□

Sample Paper 02

Class - 12th Exam - 2024 - 25

Mathematics (Code-041)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided some questions.
9. Use of calculators is not allowed.

Section - A

Section A consists of 20 questions of 1 mark each.

1. Of all the points of the feasible region, for maximum or minimum of objective functions, the point lies:

(a) inside the feasible region

(b) at the boundary line of the feasible region

(c) vertex point of the boundary of the feasible region

(d) none of the above

2. The maximum value of $y = 2x^3 - 21x^2 + 36x - 20$ is

(a) -128 (b) -126

(c) -120 (d) None of these

3. A ball thrown vertically upwards according to the formula $s = 13.8t - 4.9t^2$, where s is in metres and t is in seconds. Then its velocity at $t = 1\,\text{sec}$ is

(a) 6m/sec (b) 4 m/sec

(c) 2 m/sec (d) 8 m/sec

4. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^9 x\, dx = ?$

(a) -1 (b) 0

(c) 1 (d) $\frac{\pi}{2}$

5. What type of a relation is "Less than" in the set of real numbers?

(a) only symmetric (b) only transitive

(c) only reflexive (d) equivalence relation

6. $\tan^{-1}x + \cot^{-1}x = ?$

(a) 0 (b) 1

(c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{2}$

7. Which of the following is the unit matrix of order 3×3?

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

8. The degree of the equation $\left(\frac{d^2y}{dx^2}\right)^2 - x\left(\frac{dy}{dx}\right)^3 = y^3$ is

(a) 0 (b) 1

(c) 2 (d) 3

9. $\begin{vmatrix} \sin 20^\circ & -\cos 20^\circ \\ \sin 70^\circ & \cos 70^\circ \end{vmatrix} = ?$

(a) 1 (b) -1

(c) 0 (d) 2

10. $\int x^2 \cdot e^{x^3} dx =$

(a) $e^{x^3} + c$ (b) $\frac{1}{3}e^{x^3} + c$

(c) $e^{x^2} + c$ (d) $\frac{1}{3}e^{x^2} + c$

11. $\frac{d}{dx}[\tan x] = ?$

(a) $\sec^2 x$ (b) $\sec x$

(c) $\cot x$ (d) $-\sec^2 x$

12. The radius of a circle is increasing at the rate of 0.4 cm/s. The rate of increase of its circumference is

(a) 0.4π cm/s (b) 0.8π cm/s

(c) 0.8 cm/s (d) None of these

13. Which of the following is a homogeneous differential equation?

(a) $x^2 y dx - (x^2 + y^2) dy = 0$ (b) $(xy) dx - (x^4 + y^4) dy = 0$

(c) $(2x + y - 3) dy - (x + 2y - 3) dx = 0$ (d) $(x - y) dy = (x^2 + y + 1) dx$

14. $\int x^5 dx =$ ....

(a) $\frac{x^6}{6} + k$ (b) $\frac{x^5}{5} + k$

(c) $\frac{x^7}{7} + k$ (d) $\frac{x^8}{8} + k$

15. If l, m, n are the direction cosines of a straight line then

(a) $l^2 + m^2 - n^2 = 1$

(b) $l^2 - m^2 + n^2 = 1$

(c) $l^2 - m^2 - n^2 = 1$

(d) $l^2 + m^2 + n^2 = 1$

16. The direction ratios of the line joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) are

(a) $x_1 + x_2,\ y_1 + y_2,\ z_1 + z_2$

(b) $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

(c) $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}$

(d) $(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)$

17. $\frac{d}{dx}\left[\lim_{x \to a} \frac{x^5 - a^5}{x - a}\right] =$

(a) $5a^4$

(b) $5x^4$

(c) 1

(d) 0

18. The direction cosines of the vector $3\hat{i} - 4\hat{j} + 12\hat{k}$ is

(a) $\frac{3}{13}, \frac{4}{13}, \frac{12}{13}$

(b) $\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$

(c) $\frac{3}{\sqrt{13}}, \frac{4}{\sqrt{13}}, \frac{12}{\sqrt{13}}$

(d) $\frac{3}{\sqrt{13}}, \frac{-4}{\sqrt{13}}, \frac{12}{\sqrt{13}}$

19. **Assertion :** If $y = x^3 \cos x$, then $\frac{dy}{dx} = x^3 \sin x + 3x^2 \cos x$

Reason : $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$.

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true, but reason is not the correct explanation of assertion.

(c) Assertion is true, but reason is false.

(d) Assertion is false, but reason is true.

20. A and B are two events

Statement-I : If $P(\overline{A}) = 0.7$, $P(\overline{B}) = 0.5$ and $P(A \cup B) = 0.6$ then $P(A \cap B) = 0.2$

Reason : $P(A \cup B) + P(A \cap B) + P(\overline{A}) + P(\overline{B}) = 2.5$

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true, but reason is not the correct explanation of assertion.

(c) Assertion is true, but reason is false.

(d) Assertion is false, but reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Prove that – $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - |\vec{a}\,\vec{b}|^2$

22. If A and B are two independent events then prove that : $P(A \cup B) = 1 - P(A') \cdot P(B')$

23. Solve : $\frac{dy}{dx} + 2y\tan x = \sin x$.

OR

Show that the function $1 + 8y^2 \tan x = ay^2$ is a solution of differential equation $\cos^2 x\frac{dy}{dx} = 4y^3$

24. Find $\frac{dy}{dx}$ if $y = \cos\sqrt{\sin x}$

OR

Find $\frac{dy}{dx}$, when $x = y\log(xy)$

25. Show that the line joining the points (4, 7, 8), (2, 3, 4) is parallel to the line joining the points (2, 4, 10), (–2, –4, 2).

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. The radius of a circle is increasing uniformly at the rate of 3 cm/sec. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

27. If $f(x) = x\sin\frac{1}{x}$, when $x \neq 0$; and, $f(x) = 0$, when $x = 0$, then test the continuity of $f(x)$ at $x = 0$.

28. Find the value of $\cot^{-1}\left(\tan\frac{\pi}{7}\right)$?

OR

Prove that $4(\cot^{-1}3 + \operatorname{cosec}^{-1}\sqrt{5}) = \pi$.

29. In the set Q of all rational numbers, a binary operation $o : Q \times Q \rightarrow Q$ is defined by $o(x,y) = x\, o\, y = x + y - xy$ then show that o is commutative.

30. Find the value of p, if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = 0$

31. Find the value of x, such that $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}\begin{bmatrix} x & -5 & -1 \end{bmatrix} = 0$

OR

If $A = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 \\ 6 & 2 \end{bmatrix}$ then find $(A + B)$ and $(A - B)$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs in drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

33. Solve the following L.P. Problem graphically :

Maximise $Z = x + y$

Subject to $x - y \leq -1,\ -x + y \leq 0,\ x,\ y \geq 0$

34. Solve : $\dfrac{dy}{dx} - \dfrac{2y}{x} = y^4$

OR

Solve $y^2\,dx + (x^2 + xy)\,dy = 0$

35. Evaluate $\displaystyle\int \frac{-7x+2}{\sqrt{16x^2-9}}\,dx$

OR

Prove that $\displaystyle\int_0^{\pi/2} \log \sin x\,dx = \int_0^{\pi/2} \log \cos x\,dx = -\frac{\pi}{2}\log 2$

Section - E

Case study based questions are compulsory.

36. If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the third side of the triangle taken in opposite order and this is known as triangle law of vector addition. Based on the above information, answer the following questions.

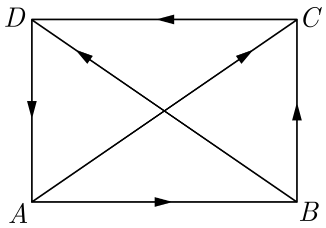
(i) If $\hat{p}$, $\hat{q}$, $\hat{r}$ are the vectors represented by the side of a triangle taken in order, then find $\vec{q} + \vec{r}$.

(ii) If $ABCD$ is a parallelogram and AC and BD are its diagonals, then find $\overrightarrow{AC} + \overrightarrow{BD}$.

(iii) If $ABCD$ is a parallelogram, where $\overrightarrow{AB} = 2\vec{a}$ and $\overrightarrow{BC} = 2\vec{b}$, then find $\overrightarrow{AC} - \overrightarrow{BD}$.

OR

(iv) If ABCD is a quadrilateral, whose diagonals are $\overrightarrow{AC}$ and $\overrightarrow{BD}$, then find $\overrightarrow{BA} + \overrightarrow{CD}$.



37. Rice is a nutritional staple food which provides instant energy as its most important component is carbohydrate (starch). On the other hand, rice is poor in nitrogenous substances with average composition of these substances being only 8 per cent and fat content or lipids only negligible, i.e., 1per cent and due to this reason it is considered as a complete food for eating. Rice flour is rich in starch and is used for making various food materials.

Two farmers Ramkishan and Gurcharan Singh cultivate only three varieties of rice namely Basmati, Permal and Naura. The sale (in ₹) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices A and B.

September Sales (in ₹)

$$\begin{array}{c} \\ A = \end{array} \begin{array}{c} \begin{matrix} \text{Basmati} & \text{Permal} & \text{Naura} \end{matrix} \\ \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Ramkishan} \\ \text{Gurcharan Singh} \end{array}$$

October Sales (in ₹)

$$\begin{array}{c} \\ B = \end{array} \begin{array}{c} \begin{matrix} \text{Basmati} & \text{Permal} & \text{Naura} \end{matrix} \\ \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Ramkishan} \\ \text{Gurcharan Singh} \end{array}$$

(i) Find the combined sales in September and October for each farmer in each variety.

(ii) Find the decrease in sales from September to October.

(iii) If both farmers receive 2% profit on gross sales, compute the profit for each farmer and for each variety sold in October.

38. Western music is a form of country music composed by and about the people who settled and worked throughout the Western United States and Western Canada. Western music celebrates the lifestyle of the cowboy on the open ranges, Rocky Mountains, and prairies of Western North America.

Western music is organised every year in the stadium that can hold 36000 spectators. With ticket price of ₹10, the average attendance has been 24000. Some financial expert estimated that price of a ticket should be determined by the function $p(x) = 15 - \frac{x}{3000}$, where x is the number of ticket sold.

Bases on the above information, answer of the following questions.

(i) Find the expression for total revenue R as a function of x.

(ii) Find the value of x for which revenue is maximum.

(iii) When the revenue is maximum, what will be the price of the ticket?

OR

(iv) How many spectators should be present to maximum the revenue?

□□□□□□□

Sample Paper 03

Class - 12th Exam - 2024 - 25

Mathematics (Code-041)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided some questions.
9. Use of calculators is not allowed.

Section - A

Section A consists of 20 questions of 1 mark each.

1. The value of $\int_{-2}^{2}(x\cos x+\sin x+1)\,dx$ is

(a) 2

(b) 0

(c) -2

(d) 4

2. $\int\frac{\sin 2x}{\sin^2 x+2\cos^2 x}\,dx$ is equal to

(a) $-\log(1+\sin^2 x)+C$

(b) $\log(1+\cos^2 x)+C$

(c) $-\log(1+\cos^2 x)+C$

(d) $\log(1+\tan^2 x)+C$

3. If $|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=144$ and $|\vec{a}|=4$, then $|\vec{b}|$ is equal to

(a) 16

(b) 8

(c) 3

(d) 12

4. If A and B are two equivalence relations defined on set C, then

(a) $A\cap B$ is an equivalence relation

(b) $A\cap B$ is not an equivalence relation

(c) $A\cap B$ is an equivalence relation

(d) $A\cap B$ is not an equivalence relation

5. If $x=e^{y+e^{y+e^{y+\dots}}}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{x}$

(b) $\frac{1-x}{x}$

(c) $\frac{x}{1+x}$

(d) None of these

6. The derivative of $\log|x|$ is

(a) $\frac{1}{x}, x > 0$ (b) $\frac{1}{|x|}, x \neq 0$

(c) $\frac{1}{x}, x \neq 0$ (d) None of these

7. $+\int_{-2}^{2} \sin x\,dx + (2+2) = 4$ Which of the following function is decreasing on $(0, \pi/2)$?

(a) $\sin 2x$ (b) $\cos 3x$

(c) $\tan x$ (d) $\cos 2x$

8. The condition that $f(x) = ax^3 + bx^2 + cx + d$ has no extreme value is

(a) $b^2 > 3ac$ (b) $b^2 = 4ac$

(c) $b^2 = 3ac$ (d) $b^2 < 3ac$

9. $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ is equal to

(a) 0 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

10. The area bounded by the curve $y = \frac{1}{2}x^2$, the X-axis and the ordinate $x = 2$ is

(a) $\frac{1}{3}$ sq unit (b) $\frac{2}{3}$ sq unit

(c) 1 sq unit (d) $\frac{4}{3}$ sq unit

11. The solution of $\frac{dy}{dx} = \frac{ax+g}{by+f}$ represents a circle, when

(a) $a = b$ (b) $a = -b$

(c) $a = -2b$ (d) $a = 2b$

12. If A and B are two symmetric matrices of same order. Then, the matrix $AB - BA$ is equal to

(a) a symmetric matrix (b) a skew-symmetric matrix

(c) a null matrix (d) the identity matrix

13. The family of curves $y = e^{a \sin x}$, where a is an arbitrary constant is represented by the differential equation

(a) $\log y = \tan x \frac{dy}{dx}$ (b) $y \log y = \tan x \frac{dy}{dx}$

(c) $y \log y = \sin \frac{dy}{dx}$ (d) $\log y = \cos x \frac{dy}{dx}$

14. The order of the differential equation whose solution is $y = a \cos x + b \sin x + ce^{-x}$, is

(a) 3 (b) 1

(c) 2 (d) 4

15. If A and B are two independent events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$, then P (neither A nor B) is equal to

(a) $\frac{2}{3}$ (b) $\frac{1}{6}$

(c) $\frac{5}{6}$ (d) $\frac{1}{3}$

16. The distance of the plane $6x-3y+2z-14=0$ from the origin is

(a) 2
(b) 1
(c) 14
(d) 8

17. If $P(A \cup B)=0.83$, $P(A)=0.3$ and $P(B)=0.6$, then the events will be

(a) dependent
(b) independent
(c) cannot say anything
(d) None of the above

18. If $\vec{x}$ and $\vec{y}$ are unit vectors and $\vec{x}\cdot\vec{y}=0$, then

(a) $|\vec{x}+\vec{y}|=1$
(b) $|\vec{x}+\vec{y}|=\sqrt{3}$
(c) $|\vec{x}+\vec{y}|=2$
(d) $|\vec{x}+\vec{y}|=\sqrt{2}$

19. Let us define $\tan^{-1}\mathrm{A}+\tan^{-1}\mathrm{B}=\tan^{-1}\left(\frac{\mathrm{A}+\mathrm{B}}{1-\mathrm{AB}}\right)$

Assertion : The value of $\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{1}{7}\right)$ is $\frac{\pi}{4}$.

Reason : If $x>0, y>0$ than $\tan^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y-x}{y+x}\right)=\frac{\pi}{4}$

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.
(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.
(c) Assertion is true, reason is false.
(d) Assertion is false, reason is true.

20. **Assertion :** If A is a matrix of order 2×2, then $|\mathrm{adj\,A}|=|\mathrm{A}|$
Reason : $|\mathrm{A}|=|\mathrm{A}^{\mathrm{T}}|$

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.
(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.
(c) Assertion is true, reason is false.
(d) Assertion is false, reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Find the differential equation representing the family of curves $y=ae^{bx+5}$, where a and b are arbitrary constants.

22. Write the value of $\int_0^1 \frac{e^x}{1+e^{2x}}dx$.

23. Write a unit vector in the direction of the sum of the vectors $\vec{a}=2\hat{i}+2\hat{j}-5\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}-7\hat{k}$.

OR

If $\vec{a}=\hat{i}-\hat{j}+7\hat{k}$ and $\vec{b}=5\hat{i}-\hat{j}+\lambda\hat{k}$, then find the value of λ, so that $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular vectors.

24. Write the vector equation of a line passing through point $(1, -1, 2)$ and parallel to the line whose equation is $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$.

25. Two groups are computing for the positions of the Board of Directors of a corporation. The probabilities that the first and second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product introduced way by the second group.

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. If $A = \{1, 2, 3\}, B = \{4, 5, 6, 7\}$ and $f = \{(1, 4), (2, 5), (3, 6)\}$ is a function from A to B. State whether f is one-one or not.

27. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$. Find a vector of magnitude 6 units, which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

28. Evaluate $\int_0^{\pi/2} x^2 \sin x \, dx$.

OR

Evaluate $\int_{\pi/4}^{\pi/2} \cos 2x \cdot \log(\sin x) \, dx$.

29. Solve for x, $\cos^{-1} x + \sin^{-1}\left(\frac{x}{2}\right) = \frac{\pi}{6}$.

30. Evaluate $\int \frac{x^2 + x + 1}{(x^2 + 1)(x + 2)} dx$.

31. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.

OR

If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then find the value of k.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{p}$, which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p} \cdot \vec{c} = 18$.

33. Find the particular solution of the differential equation satisfying the given condition.
$x^2 \, dy + (xy + y^2) \, dx = 0$, when $y(1) = 1$

OR

Find the particular solution of the differential equation $x\frac{dy}{dx} - y + x \operatorname{cosec}\left(\frac{y}{x}\right) = 0$

34. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$, then find the values of a and b.

OR

Find the adjoint of the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that $A(adj\ A) = |A| I_3$.

35. If $x = \cos t + \log \tan\left(\frac{t}{2}\right)$, $y = \sin t$, then find the values of $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$.

OR

Find the values of a and b, if the function f defined by $f(x) = \begin{cases} x^2 + 3x + a, & x \leq 1 \\ bx + 2, & x > 1 \end{cases}$ is differentiable at $x = 1$.

Section - E

Case study based questions are compulsory.

36. Vitamins are nutritional substances which you need in small amounts in your diet. Vitamins A and E are fat-soluble vitamins, meaning they're stored in your body's fat cells, but they need to have their levels topped up regularly. Vitamin C is a water-soluble vitamin found in citrus and other fruits and vegetables, and also sold as a dietary supplement. It is used to prevent and treat scurvy. Vitamin C is an essential nutrient involved in the repair of tissue, the formation of collagen, and the enzymatic production of certain neurotransmitters.

A dietician wishes to mix two types of foods in such a way that the vitamin contents of mixture contains atleast 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C, while food II contains 1 unit per kg of vitamin A an 2 units per kg of vitamin C. It costs Rs. 30 per kg to purchase food I and Rs. 42 per kg to purchase food II.

(i) Formulate above as an LPP and solve it graphically.

(ii) Find the minimum cost of such a mixture.

37. Chemical reaction, a process in which one or more substances, the reactants, are converted to one or more different substances, the products. Substances are either chemical elements or compounds. A chemical reaction rearranges the constituent atoms of the reactants to create different substances as products.

In a certain chemical reaction, a substance is converted into another substance at a rate proportional to the square of the amount of the first substance present at any time t. Initially $(t=0)$ 50 g of the first substance was present; 1 hr later, only 10 g of it remained.

(i) Find an expression that gives the amount of the first substance present at any time t.

(ii) What is the amount present after 2 hr?

38. An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20% of the population is accident prone.

On the basis of above information, answer the following questions.

(i) What is the probability that a new policyholder will have an accident within a year of purchasing a policy?

(ii) Suppose that a new policy holder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?

□□□□□□□

Sample Paper 04

Class - 12th Exam - 2024 - 25

Mathematics (Code-041)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided some questions.
9. Use of calculators is not allowed.

Section - A

Section A consists of 20 questions of 1 mark each.

1. If $\int_0^a f(2a-x)\,dx = m$ and $\int_0^a f(x)\,dx = n$, then $\int_0^{2a} f(x)\,dx$ is equal to

(a) $2m+n$ (b) $m+2n$

(c) $m-n$ (d) $m+n$

2. A coin and six faced die, both unbiased, are thrown simultaneously. The probability of getting a head on the coin and an odd number on the die is

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$

(c) $\frac{1}{4}$ (d) $\frac{2}{3}$

3. The function $f(x) = x^2 e^{-x}$ is strictly increases in the interval

(a) $(0, 2)$ (b) $(0, \infty)$

(c) $(-\infty, 0] \cup [2, \infty)$ (d) none of theses

4. $\int \frac{\sec^2(\sin^{-1}x)}{\sqrt{1-x^2}}\,dx$ is equal to

(a) $\sin(\tan^{-1}x) + C$ (b) $\tan(\sec^{-1}x) + C$

(c) $\tan(\sin^{-1}x) + C$ (d) $-\tan(\cos^{-1}x) + C$

5. If R is a relation on the set N, defined by $\{(x, y): 2x - y = 10\}$, then R is

(a) reflexive (b) symmetric

(c) transitive (d) None of the above

6. If $\sin^{-1}x = \theta + \beta$ and $\sin^{-1}y = \theta - \beta$, then $1 + xy$ is equal to

(a) $\sin^2\theta + \sin^2\beta$ (b) $\sin^2\theta + \cos^2\beta$

(c) $\cos^2\theta + \cos^2\beta$ (d) none of these

7. Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}, \vec{b} = \hat{j}$. The value of $\vec{c}$ for which $\vec{a}, \vec{b}, \vec{c}$ form a right handed system is

(a) $\vec{0}$ (b) $z\hat{i} - x\hat{k}$

(c) $-z\hat{i} + x\hat{k}$ (d) $y\hat{j}$

8. If the function $f(x) = kx^3 - 9x^2 + 9x + 3$ is monotonically increasing in every interval, then

(a) $k < 3$ (b) $k \leq 3$

(c) $k > 3$ (d) $k \geq 3$

9. If $f(x) = \begin{cases} \dfrac{3\sin \pi x}{5x}, & x \neq 0 \\ 2k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

(a) $\dfrac{\pi}{10}$ (b) $\dfrac{3\pi}{10}$

(c) $\dfrac{3\pi}{2}$ (d) $\dfrac{3\pi}{5}$

10. The domain of the function $f(x) = \sqrt{\cos x}$ is

(a) $\left[\dfrac{3\pi}{2}, 2\pi\right]$ (b) $\left[0, \dfrac{\pi}{2}\right]$

(c) $[-\pi, \pi]$ (d) $\left[0, \dfrac{\pi}{2}\right] \cup \left[\dfrac{3\pi}{2}, 2\pi\right]$

11. If $f(x) = \begin{cases} \dfrac{\log x}{x-1}, & \text{if } x \neq 1 \\ k, & \text{if } x = 1 \end{cases}$ is continuous at $x = 1$, then the value of k is

(a) 0 (b) -1

(c) 1 (d) e

12. Inverse of the matrix $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ is

(a) $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ (b) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

(c) $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ (d) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

13. Order of the differential equation of the family of all concentric circles centred at (h, k), is

(a) 2 (b) 3

(c) 1 (d) 4

14. If m and n are the order and degree of the differential equation $\left(\dfrac{d^2y}{dx^2}\right)^5 + 4\dfrac{\left(\dfrac{d^2y}{dx^2}\right)^3}{\dfrac{d^3y}{dx^3}} + \dfrac{d^3y}{dx^3} = x^2 - 1$, then

(a) $m = 3, n = 2$ (b) $m = 3, n = 3$

(c) $m = 3, n = 5$ (d) $m = 3, n = 1$

15. The vectors $\vec{a} = 2\hat{i} - 3\hat{j}$ and $\vec{b} = -4\hat{i} + 6\hat{j}$ are

(a) coincident
(b) parallel
(c) perpendicular
(d) neither parallel nor perpendicular

16. The area bounded by $y = \log x$, X-axis and ordinates $x = 1$, $x = 2$ is

(a) $\frac{1}{2}(\log 2)^2$
(b) $\log(2/e)$
(c) $\log(4/e)$
(d) $\log 4$

17. A straight line which makes an angle of 60° with each of y and z axes, inclined with x-axis at an angle of

(a) 30°
(b) 45°
(c) 75°
(d) 60°

18. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(\frac{A}{B}) = \frac{1}{2}$ and $P(\frac{B}{A}) = \frac{2}{3}$. Then, $P(B)$ is equal to

(a) $\frac{1}{2}$
(b) $\frac{1}{6}$
(c) $\frac{1}{3}$
(d) $\frac{2}{3}$

19. **Assertion:** The equation of curve passing through (3, 9) which satisfies differential equation $\frac{dy}{dx} = x + \frac{1}{x^2}$ is $6xy = 3x^3 + 29x - 6$

Reason: The solution of differential equation $\left(\frac{dy}{dx}\right)^2 - \left(\frac{dy}{dx}\right)(e^x + e^{-x}) + 1 = 0$ is $y = c_1 e^x + c_2 e^{-x}$.

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.
(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.
(c) Assertion is true; Reason is false.
(d) Assertion is false; Reason is true.

20. Let A be a 2×2 matrix.
Assertion: adj (adj A) $= A$.
Reason: $|\text{adj } A| = |A|$.

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.
(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.
(c) Assertion is true; Reason is false.
(d) Assertion is false; Reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. A line passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is the direction of the vector $\hat{i} + \hat{j} - 2\hat{k}$. Find the equation of the line in cartesian form.

22. State the reason for the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ not to be transitive.

23. Write the vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 units.

OR

Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$.

24. Evaluate $\int (1 - x)\sqrt{x}\, dx$.

OR

Given, $\int e^x (\tan x + 1) \sec x\, dx = e^x f(x) + C$. Write $f(x)$ satisfying above.

25. If $P(not\ A) = 0.7$, $P(B) = 0.7$ and $P(B/A) = 0.5$, then find $P(A/B)$.

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Find $\int \frac{dx}{5 - 8x - x^2}$.

27. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a} + \vec{b}| = 13$ and $|\vec{a}| = 5$, then find the value of $|\vec{b}|$.

28. The volume of a cube is increasing at the rate of 8 cm3/s. How fast is the surface area increasing when the length of its edge is 12 cm?

OR

The side of an equilateral triangle is increasing at the rate of 2 cm/s. At what rate is its area increasing, when the side of the triangle is 20 cm?

29. If $A = \begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$, then write the cofactor of the element a_{21} of its 2nd row.

30. Write the value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$.

31. If $y = x\cos(a + y)$, then show that $\frac{dy}{dx} = \frac{\cos^2(a + y)}{\cos a}$ Also, show that $\frac{dy}{dx} = \cos a$, when $x = 0$.

OR

If $x = a\sec^3\theta$ and $y = a\tan^3\theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{3}$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Find the values of p, so that the lines

$$l_1 : \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2} \text{ and } l_2 : \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

are perpendicular to each other. Also, find the equation of a line passing through a point $(3, 2, -4)$ and parallel to line l_1.

33. Show that the differential equation $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ is homogeneous and also solve it.

34. Find $\int \frac{2\cos x}{(1-\sin x)(2-\cos^2 x)} dx$.

OR

Find $\int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)} dx$.

35. A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time, B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one items is chosen at random from this and is found to be defective. What is the probability that it was produced by A?

OR

An insurance company insured 2000 scooter driver, 4000 car drivers and 6000 truck drivers. The probabilities of an accident for them are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver or a car driver?

Section - E

Case study based questions are compulsory.

36. The Indian toy industry is estimated to be worth US$1.5 billion, making up 0.5% of the global market share. The toy manufacturers in India can mostly be found in NCR, Mumbai, Karnataka, Tamil Nadu, and several smaller towns and cities across central states such as Chhattisgarh and Madhya Pradesh. The sector is fragmented with 90% of the market being unorganised. The toys industry has been predicted to grow to US$2-3 billion by 2024. The Indian toy industry only represents 0.5% of the global industry size indicating a large potential growth opportunity for Indian consumer product companies who will develop exciting innovations to deliver international quality standards at competitive prices.

Fisher Price is a leading toy manufacturer in India. Fisher Price produces x set per week at a total cost of $\frac{1}{25}x^2 + 3x + 100$. The produced quantity for his market is $x = 75 - 3p$ where p is the price set.

(i) Show that the maximum profit is obtained when about 30 toys are produced per week.

(ii) What is the price at maximum profit?

37. A manufacturing company has two service departments, S_1, S_2 and four production departments P_1, P_2, P_3 and P_4.

Overhead is allocated to the production departments for inclusion in the stock valuation. The analysis of benefits received by each department during the last quarter and the overhead expense incurred by each department were:

Service Department	**Percentages to be allocated to departments**					
	S_1	S_2	P_1	P_2	P_3	P_4
S_1	0	20	30	25	15	10
S_2	30	0	10	35	20	5
Direct overhead expense ₹ '000	20	40	25	30	20	10

You are required to find out following using matrix method.

(i) Express the total overhead of the service departments in the form of simultaneous equations.

(ii) Express these equations in a matrix form and solve for total overhead of service departments using matrix inverse method.

(iii) Determine the total overhead to be allocated from each of S_1 and S_2 to the production department.

38. A craftswoman produces two products: floor lamps and table lamps. Production of one floor lamp requires 75 minutes of her labor and materials that cost \$25. Production of one table lamp requires 50 minutes of labor, and the materials cost \$20. The craftswoman wishes to work no more than 40 hours each week, and her financial resources allow her to pay no more than \$900 for materials each week.

(i) If she can sell as many lamps as she can make and if her profit is \$39 per floor lamp and \$33 per table lamp, how many floor lamps and how many table lamps should she make each week to maximize her weekly profit?

(ii) What is that maximum profit?

□□□□□□□□

Sample Paper 05

Class - 12th Exam - 2024 - 25

Mathematics (Code-041)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided some questions.
9. Use of calculators is not allowed.

Section - A

Section A consists of 20 questions of 1 mark each.

1. The point of intersection of the lines $\vec{r}\times\vec{a}=\vec{b}\times\vec{a}$ and $\vec{r}\times\vec{b}=\vec{a}\times\vec{b}$ is

(a) $\vec{a}$ (b) $\vec{b}$

(c) $\vec{a}+\vec{b}$ (d) $\vec{a}-\vec{b}$

2. The foot of the perpendicular from (0, 2, 3) to the line $\frac{x+3}{5}=\frac{y=1}{2}=\frac{z+4}{3}$ is

(a) $(-2,3,4)$ (b) $(2,-1,3)$

(c) $(2,3,-1)$ (d) $(3,2,-1)$

3. Which of the following function is decreasing on $(0,\pi/2)$?

(a) $\sin 2x$ (b) $\cos 3x$

(c) $\tan x$ (d) $\cos 2x$

4. $2\sin^{-1}x=\sin^{-1}(2x\sqrt{1-x^2})$ holds good for all

(a) $|x|\leq 1$ (b) $1\geq x\geq 0$

(c) $|x|\leq\frac{1}{\sqrt{2}}$ (d) none of these

5. If $f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2}, x\neq 0$, then $f(x)$ is equal to

(a) x^2-2 (b) $x+2$

(c) x^2+2 (d) $2x^2-5$

6. If A and B are two symmetric matrices of same order. Then, the matrix $AB-BA$ is equal to

(a) a symmetric matrix (b) a skew-symmetric matrix

(c) a null matrix (d) the identity matrix

7. If $y = \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ is

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) 1 (d) -1

8. The general solution of the differential equation $\frac{dy}{dx} + \sin\frac{x+y}{2} = \sin\frac{x-y}{2}$ is

(a) $\log\tan\left(\frac{y}{2}\right) = c - 2\sin x$ (b) $\log\tan\left(\frac{y}{4}\right) = c - 2\sin\frac{x}{2}$

(c) $\log\tan\left(\frac{y}{2}+\frac{\pi}{4}\right) = c - 2\sin x$ (d) $\log\tan\left(\frac{y}{4}+\frac{\pi}{4}\right) = c - 2\sin\left(\frac{x}{2}\right)$

9. The degree of the differential equation satisfying $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$

(a) 1 (b) 2

(c) 3 (d) 4

10. Let $f(x) = \tan x - 4x$, then in the interval $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$, $f(x)$ is

(a) a decreasing function (b) an increasing function

(c) a constant function (d) none of these

11. The value of $\int_0^1 \frac{dx}{e^x + e}$ is

(a) $\frac{1}{e}\log\left(\frac{1+e}{2}\right)$ (b) $\log\left(\frac{1+e}{2}\right)$

(c) $\frac{1}{e}\log(1+e)$ (d) $\log\left(\frac{2}{1+e}\right)$

12. If $A = \{1,2,3,4\}$ and $B = \{1,2,3,4,5,6\}$ are two sets and function $f : A \to B$ is defined by $f(x) = x+2$, $\forall x \in A$, then the function f is

(a) bijective (b) onto

(c) one-one (d) many-one

13. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$, then $A \cdot (adj\ A)$ is equal to

(a) A (b) $|A|$

(c) $|A| \cdot I$ (d) None of these

14. The area of enclosed by $y = 3x - 5$, $y = 0$, $x = 3$ and $x = 5$ is

(a) 12 sq units (b) 13 sq units

(c) $13\frac{1}{2}$ sq units (d) 14 sq units

15. Solution of the equation $x\frac{dy}{dx} = y + x\tan\frac{y}{x}$ is

(a) $\sin\frac{x}{y} = cx$ (b) $\sin\frac{y}{x} = cx$

(c) $\sin\frac{x}{y} = cy$ (d) $\sin\frac{y}{x} = cy$

16. If $x = e^{y+e^{y+e^{y+\dots}}}$, then $\dfrac{dy}{dx}$ is equal to

(a) $\dfrac{1}{x}$ (b) $\dfrac{1-x}{x}$

(c) $\dfrac{x}{1+x}$ (d) None of these

17. If A and B are two independent events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$, then P (neither A nor B) is equal to

(a) $\frac{2}{3}$ (b) $\frac{1}{6}$

(c) $\frac{5}{6}$ (d) $\frac{1}{3}$

18. Objective function of a linear programming problem is

(a) a constraint (b) a function to be optimized

(c) a relation between the variables (d) none of the above

19. **Assertion:** The pair of lines given by $\vec{r} = \hat{i} - \hat{j} + \lambda(2\hat{i} + k)$ and $\vec{r} = 2\hat{i} - k + \mu(\hat{i} + \hat{j} - \hat{k})$ intersect.
Reason: Two lines intersect each other, if they are not parallel and shortest distance $= 0$.

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

20. **Assertion:** $\int_0^{2\pi} \sin^3 x \, dx = 0$

Reason: $\sin^3 x$ is an odd function

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. The x-coordinate of point on the line joining the points $P(2,2,1)$ and $Q(5,1,-2)$ is 4. Find its z -coordinate.

22. Let R is the equivalence relation in the set $A = \{0,1,2,3,4,5\}$ given by $R = \{(a,b): 2 \text{ divides } (a-b)\}$. Write the equivalence class [0].

23. Evaluate $\int \dfrac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} dx$.

OR

Evaluate $\int \dfrac{\cos\sqrt{x}}{\sqrt{x}} dx$.

24. Write the direction ratios of the vector $3\vec{a} + 2\vec{b}$, where $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$.

OR

Find the unit vector in the direction of the sum of vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $4\hat{i} - 3\hat{j} + 2\hat{k}$.

25. If $P(A) = \frac{1}{12}$, $P(B) = \frac{5}{12}$ and $P\left(\frac{B}{A}\right) = \frac{1}{15}$, then find $P(A \cup B)$

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. If $e^y(x+1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

27. If $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$, then find the value of λ, so that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular vectors.

28. Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$.

OR

Evaluate $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$.

29. If $\begin{vmatrix} 3x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then write the value of x.

30. Write the value of $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$.

31. The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2 cm.

OR

Find the intervals in which the function $f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$ is

(i) strictly increasing

(ii) strictly decreasing.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Evaluate $\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2 \sin^2 x} dx$.

33. Find the values of p, so that the lines $l_1 : \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$ and $l_2 : \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other. Also, find the equation of a line passing through a point $(3, 2, -4)$ and parallel to line l_1.

34. Solve the differential equation $x \log|x|\frac{dy}{dx} + y = \frac{2}{x}\log|x|$.

OR

Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ given that $y = 1$, when $x = 0$.

35. Maximize $Z = -x + 2y$, subject to the constraints : $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$

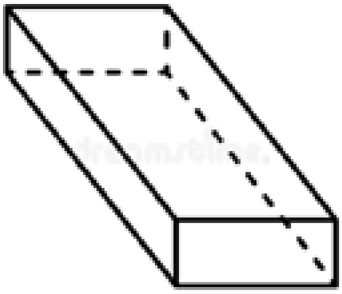
OR

Maximize $Z = x + y$, subject to $x - y \leq -1$, $x + y \leq 0$, x, $y \geq 0$.

Section - E

Case study based questions are compulsory.

36. Parallelepiped is the Greek word, which essentially means the object that has a parallel plane. Principally, the Parallelepiped is framed by the six parallelogram sides which bring about the prism or the 3D figure, and it consists of the parallelogram base. It can be categorized as anything but the polyhedron, where 3 sets of the parallel faces are made to combine for framing a three-dimensional (3D) shape that has six faces. The cube, cuboid, and rhomboid are the three exceptional cases. The Rectangular Parallelepiped consists of six faces in a rectangular shape.



The sum of the surface area of a rectangular parallelopied with sides of x, $2x$ and $\frac{x}{3}$ and a shape of radius of y is given to be constant.
On the basis of above information, answer the following questions.
(i) If S is the constant, then find the relation between S, x and y.
(ii) If the combined volume is denoted by V , then find the relation between V, x and y.
(iii) Find the relation between x and y when the volume V is minimum.

OR

(iv) If at $x = 3y$, volume V is minimum, then find the value of minimum volume and the value of S.

37. A manufacturing company has two service departments, S_1, S_2 and four production departments P_1, P_2, P_3 and P_4.
Overhead is allocated to the production departments for inclusion in the stock valuation. The analysis of benefits received by each department during the last quarter and the overhead expense incurred by each department were :

Service Department	**Percentages to be allocated to departments**					
	S_1	S_2	P_1	P_2	P_3	P_4
S_1	0	20	30	25	15	10
S_2	30	0	10	35	20	5
Direct overhead expense ₹ '000	20	40	25	30	20	10

You are required to find out following using matrix method.

(i) Express the total overhead of the service departments in the form of simultaneous equations.

(ii) Express these equations in a matrix form and solve for total overhead of service departments using matrix inverse method.

(iii) Determine the total overhead to be allocated from each of S_1 and S_2 to the production department.

38. Rajneesh do outsourcing work for companies and runs a form processing agency. He collect form from different office and then extract data and record data on computer. In his office three employees Vikas, Sarita and Ishaan process incoming copies of a form. Vikas process 50% of the forms. Sarita processes 20% and Ishaan the remaining 30% of the forms. Vikas has an error rate of 0.06, Sarita has an error rate of 0.04 and Ishaan has an error rate of 0.03.

Based on the above information answer the following questions.

(i) The total probability of committing an error in processing the form.

(ii) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is not processed by Vikas.

□□□□□□□

Sample Paper 06

Class - 12th Exam - 2024 - 25

Mathematics (Code-041)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided some questions.
9. Use of calculators is not allowed.

Section - A

Section A consists of 20 questions of 1 mark each.

1. The area bounded by the parabola $y^2 = 8x$ and its latus rectum is

(a) 16/3 sq units (b) 32/3 sq units

(c) 8/3 sq units (d) 64/3 sq units

2. If $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|$, then $\vec{a}, \vec{b}$ are

(a) perpendicular (b) like parallel

(c) unlike parallel (d) coincident

3. Let $f(x) = x^3 + \frac{3}{2}x^2 + 3x + 3$, then $f(x)$ is

(a) am even function (b) an odd function

(c) an increasing function (d) a decreasing function

4. If $P = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix}$ and $Q = PP^T$, then the value of Q is

(a) 2 (b) -2

(c) 1 (d) 0

5. $\int \frac{dx}{x(x^7+1)}$ is equal to

(a) $\log\left(\frac{x^7}{x^7+1}\right) + C$ (b) $\frac{1}{7}\log\left(\frac{x^7}{x^7+1}\right) + C$

(c) $\log\left(\frac{x^7+1}{x^7}\right) + C$ (d) $\frac{1}{7}\log\left(\frac{x^7+1}{x^7}\right) + C$

6. The value of $\int_0^1 \frac{dx}{e^x + e}$ is

(a) $\frac{1}{e}\log\left(\frac{1+e}{2}\right)$

(b) $\log\left(\frac{1+e}{2}\right)$

(c) $\frac{1}{e}\log(1+e)$

(d) $\log\left(\frac{2}{1+e}\right)$

7. If $A = \{1,3,5,7\}$ and $B = \{1,2,3,4,5,6,7,8\}$, then the number of one-one function from A into B is

(a) 1340

(b) 1860

(c) 1430

(d) 1680

8. The relation $\operatorname{cosec}^{-1}\left(\frac{x^2+1}{2x}\right) = 2\cot^{-1}x$ is valid for

(a) $x \geq 1$

(b) $x \geq 0$

(c) $|x| \geq 1$

(d) none of these

9. If $x = e^{y+e^{y+e^{y+\dots}}}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{x}$

(b) $\frac{1-x}{x}$

(c) $\frac{x}{1+x}$

(d) None of these

10. The point of discontinuous of $\tan x$ are

(a) $n\pi,\ n \in I$

(b) $2n\pi,\ n \in I$

(c) $(2n+1)\frac{\pi}{2},\ n \in I$

(d) None of these

11. The length of the largest interval in which the function $3\sin x - 4\sin^3 x$ is increasing, is

(a) $\frac{\pi}{3}$

(b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{2}$

(d) π

12. Two dice are thrown n times in succession. The probability of obtaining a doublet six atleast once is

(a) $\left(\frac{1}{36}\right)^n$

(b) $1-\left(\frac{35}{36}\right)^n$

(c) $\left(\frac{1}{12}\right)^n$

(d) None of these

13. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{d^2y}{dx^2}\right)$

(a) 1

(b) 3

(c) 2

(d) none of these

14. The solution of the differential equation $x^2\frac{dy}{dx} - xy = 1 + \cos\frac{y}{x}$ is

(a) $\tan\frac{y}{2x} = c - \frac{1}{2x^2}$

(b) $\tan\frac{y}{x} = c + \frac{1}{x}$

(c) $\cos\frac{y}{x} = 1 + \frac{c}{x}$

(d) $x^2 = (c + x^2)\tan\frac{y}{x}$

15. Let G be the centroid of a triangle ABC and O be any other point, then $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}$ is equal to

(a) $\vec{0}$

(b) $\overrightarrow{OG}$

(c) $3\overrightarrow{OG}$

(d) none of these

16. The projections of a line segment on x, y, z axes are 12, 4, 3. The length and the direction cosines of the line segment are

(a) $13, < \frac{12}{13}, \frac{4}{13}, \frac{3}{13} >$

(b) $19, < \frac{12}{19}, \frac{4}{19}, \frac{3}{19} >$

(c) $11, < \frac{12}{11}, \frac{14}{11}, \frac{3}{11} >$

(d) none of these

17. If A and B are mutually exclusive events with $P(B) \neq 1$, then $P(A/\overline{B})$ is equal to (here, $\overline{B}$ is the complement of the event B)

(a) $\frac{1}{P(B)}$

(b) $\frac{1}{1-P(B)}$

(c) $\frac{P(A)}{P(B)}$

(d) $\frac{P(A)}{1-P(B)}$

18. Equation to the curve through (2,1) whose slope at the point (x, y) is $\frac{x^2+y^2}{2xy}$, is

(a) $2(x^2-y^2)=3x$

(b) $2(y^2-x^2)=6y$

(c) $x(x^2-y^2)=6$

(d) none of these

19. **Assertion:** The relation R in a set $A=\{1,2,3,4\}$ defined by $R=\{(x,y): 3x-y=0)\}$ have the domain $=\{1,2,3,4\}$ and range $=\{3,6,9,12\}$

Reason: Domain & range of the relation (R) is respectively the set of all first & second entries of the distinct ordered pair of the relation.

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

20. **Assertion:** If $A=\begin{bmatrix}\sqrt{3} & 1 \\ 2 & 2\end{bmatrix}$ and $b=\begin{bmatrix}x & y & z \\ a & b & c\end{bmatrix}$, then orders of $(A+B)$ is 2×3

Reason: If $[aij]$ and $[bij]$ are two matrics of the same order, then order of $A+B$ is the same as the order of A or B

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. The probability distribution of a random variable X is given below

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
$P(X)$	p	$2p$	$3p$	$4p$	$5p$	$7p$	$8p$	$9p$	$10p$	$11p$	$12p$

What is the value of p ?

22. Show that the relation R on IR defined as $R = \{(a, b) : (a \leq b)\}$, is reflexive and transitive but not symmetric.

23. Write the value of $\int \frac{2 - 3\sin x}{\cos^2 x} dx$.

OR

Write the value of $\int \sec x(\sec x + \tan x)\, dx$.

24. If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, find the angle between $\vec{a}$ and $\vec{b}$.

OR

Write the value of λ, so that the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular to each other.

25. The equation of a line is $5x - 3 = 15y + 7 = 3 = 3 - 10z$ Write the direction cosines of the line.

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Prove that $\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\}$ $\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right\} = \frac{2b}{a}$

27. If $\begin{vmatrix} 3x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then write the value of x.

28. Show that $y = \log(1 + x) - \frac{2x}{2 + x}$, $x > -1$ is an increasing function of x, throughout its domain.

OR

Find the intervals in which the function given by $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$ is

(i) increasing (ii) decreasing.

29. If $y = (\sin x)^x + \sin^{-1}\sqrt{x}$, then find $\frac{dy}{dx}$.

30. If θ is the angle between two vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$, find $\sin\theta$.

31. Evaluate $\int_0^{\pi} |x^3 - x|\, dx$.

OR

Evaluate $\int_{-2}^{2} \frac{x^2}{1 + 5^x} dx$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Solve the following differential equation $\operatorname{cosec} x \log|y| \frac{dy}{dx} + x^2 y^2 = 0$.

OR

Solve the following differential equation. $x \cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x;\ x \neq 0$

33. Find the values of p, so that the lines

$$l_1 : \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2} \text{ and } l_2 : \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

are perpendicular to each other. Also, find the equation of a line passing through a point $(3, 2, -4)$ and parallel to line l_1.

34. Maximize $Z = 3x + 4y$, subject to the constraints; $x + y \leq 4$, $x \geq 0$, $y \geq 0$.

OR

Minimize $Z = -3x + 4y$ subject to the constraints
$x + 2y \leq 8$, $3x + 2y \leq 12$, $x \geq 0$, $y \geq 0$

35. Evaluate $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$.

Section - E

Case study based questions are compulsory.

36. Mahesh runs a form processing agency. He collect form from different office and then extract data and record data on computer. In his office three employees Vikas, Sarita and Ishaan process incoming copies of a form. Vikas process 50% of the forms. Sarita processes 20% and Ishaan the remaining 30% of the forms. Vikas has an error rate of 0.06, Sarita has an error rate of 0.04 and Ishaan has an error rate of 0.03.

Based on the above information answer the following questions.

(i) The total probability of committing an error in processing the form.

(ii) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is not processed by Vikas.

37. RK Verma is production analysts of a ready-made garment company. He has to maximize the profit of company using data available. He find that $P(x) = -6x^2 + 120x + 25000$ (in Rupee) is the total profit function of a company where x denotes the production of the company.

Based on the above information, answer the following questions.

(i) Find the profit of the company, when the production is 3 units.

(ii) Find $P'(5)$

(iii) Find the interval in which the profit is strictly increasing.

OR

(iv) Find the production, when the profit is maximum.

38. Publishing is the activity of making information, literature, music, software and other content available to the public for sale or for free. Traditionally, the term refers to the creation and distribution of printed works, such as books, newspapers, and magazines.

NODIA Press is a such publishing house having two branch at Jaipur. In each branch there are three offices. In each office, there are 2 peons, 5 clerks and 3 typists. In one office of a branch, 5 salesmen are also working. In each office of other branch 2 head-clerks are also working. Using matrix notations find:

(i) the total number of posts of each kind in all the offices taken together in each branch.

(ii) the total number of posts of each kind in all the offices taken together from both branches.

Sample Paper 07

Class - 12th Exam - 2024 - 25

Mathematics (Code-041)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided some questions.
9. Use of calculators is not allowed.

Section - A

Section A consists of 20 questions of 1 mark each.

1. The function $f(x) = x^2 - 2x$ is increasing in the interval

(a) $x \neq -1$
(b) $x \geq -1$
(c) $x \neq 1$
(d) $x \geq 1$

2. If $x = \dfrac{2at}{1+t^3}$ and $y = \dfrac{2at^2}{(1+t^3)^2}$, then $\dfrac{dy}{dx}$ is equal to

(a) ax
(b) a^2x^2
(c) $\dfrac{x}{a}$
(d) $\dfrac{x}{2a}$

3. Which of the following is correct for the function $f(x) = \sin 2x - 1$ at the point $x = 0$ and $x = \pi$

(a) Continuous at $x = 0,\ \pi$
(b) Discontinuous at $x = 0$ but continuous at $x = \pi$
(c) Continuous at $x = 0$ but discontinuous at $x = \pi$
(d) Discontinuous at $x = 0,\ \pi$

4. The minimum value of $f(x) = \sin x \cos x$ is

(a) $\dfrac{1}{2}$
(b) $-\dfrac{1}{2}$
(c) 0
(d) 5

5. If $R = \{(3,3),(6,6),(9,9),(12,12),(6,12),(3,9),(3,12),(3,6)\}$ is a relation on the set $A = \{3,6,9,12\}$. Then, the relation is

(a) an equivalence relation
(b) reflexive and symmetric
(c) reflexive and symmetric
(d) only reflexive

6. If $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.3$, then $P\left(\frac{A'}{B}\right)$ is equal to

(a) $\frac{1}{3}$
(b) $\frac{1}{2}$
(c) $\frac{2}{3}$
(d) $\frac{3}{4}$

7. The value of $\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]$ is

(a) $\frac{24}{25}$
(b) $-\frac{24}{25}$
(c) $\frac{7}{25}$
(d) none of these

8. $\int \frac{\sin 2x}{\sin^2 x + 2\cos^2 x}\,dx$ is equal to

(a) $-\log(1+\sin^2 x) + C$
(b) $\log(1+\cos^2 x) + C$
(c) $-\log(1+\cos^2 x) + C$
(d) $\log(1+\tan^2 x) + C$

9. $\int_0^{\pi} \sqrt{\frac{1+\cos 2x}{2}}\,dx$ is equal to

(a) 0
(b) 2
(c) 4
(d) -2

10. If $AB = A$ and $BA = B$, then B^2 is equal to

(a) B
(b) A
(c) $-B$
(d) B^3

11. The solution of $e^{dy/dx} = x+1, y(0) = 3$, is

(a) $y = x\log x - x + 2$
(b) $y = (x+1)\log(x+1) - x + 3$
(c) $y = (x+1)\log(x+1) + x + 3$
(d) $y = x\log x + x + 3$

12. The area of enclosed by $y = 3x - 5$, $y = 0$, $x = 3$ and $x = 5$ is

(a) 12 sq units
(b) 13 sq units
(c) $13\frac{1}{2}$ sq units
(d) 14 sq units

13. $(x^2 + xy)\,dy = (x^2 + y^2)\,dx$ is

(a) $\log x = \log(x-y) + \frac{y}{x} + c$
(b) $\log x = 2\log(x-y) + \frac{y}{x} + c$
(c) $\log x = \log(x-y) + \frac{x}{y} + c$
(d) none of the above

14. The general solution of the differential equation $x(1+y^2)\,dx + y(1+x^2)\,dy = 0$ is

(a) $(1+x^2)(1+y^2) = 0$

(b) $(1+x^2)(1+y^2) = c$

(c) $(1+y^2) = c(1+x^2)$

(d) $(1+x^2) = c(1+y^2)$

15. If $\vec{a}$ and $\vec{b}$ are position vectors of A and B respectively, then the position vector of a point C in AB produced such that $\overrightarrow{AC} = 3\overrightarrow{AB}$, is

(a) $3\vec{a} - \vec{b}$

(b) $3\vec{b} - \vec{a}$

(c) $3\vec{a} - 2\vec{b}$

(d) $3\vec{b} - 2\vec{a}$

16. If $\vec{a} = \hat{i} + \hat{j}, \vec{b} = 2\hat{j} - \hat{k}$ and $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$, $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$, then $\frac{\vec{r}}{|\vec{r}|}$ is equal to

(a) $\frac{1}{\sqrt{11}}(\hat{i} + 3\hat{j} - \hat{k})$

(b) $\frac{1}{\sqrt{11}}(\hat{i} - 3\hat{j} + \hat{k})$

(c) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$

(d) none of these

17. The foot of the perpendicular from (0, 2, 3) to the line $\frac{x+3}{5} = \frac{y=1}{2} = \frac{z+4}{3}$ is

(a) $(-2, 3, 4)$

(b) $(2, -1, 3)$

(c) $(2, 3, -1)$

(d) $(3, 2, -1)$

18. A mapping $f: n \to N$, where N is the set of natural numbers is define as

$$f(n) = \begin{cases} n^2, & \text{for } n \text{ odd} \\ 2n+1, & \text{for } n \text{ even} \end{cases} \text{ for } n \in N. \text{ Then, } f \text{ is}$$

(a) Surjective but not injective

(b) Injective but not surjective

(c) Bijective

(d) neither injective nor surjective

19. Let A and B be two events associated with an experiment such that $P(A \cap B) = P(A)P(B)$

Assertion: $P(A\,|\,B) = P(A)$ and $P(B\,|\,A) = P(B)$

Reason: $P(A \cup B) = P(A) + P(B)$

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

20. For any square matrix A with real number entries consider the following statements.

Assertion: $A + A'$ is a symmetric matrix.

Reason: $A - A'$ is a skew-symmetric matrix.

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. What are the direction consines of a line which makes equal angles with the coordinate axes?

22. If $A = \{1, 2, 3\}, B = \{4, 5, 6, 7\}$ and $f = \{(1, 4), (2, 5), (3, 6)\}$ is a function from A to B. State whether f is one-one or not.

23. Two groups are computing for the positions of the Board of Directors of a corporation. The probabilities that the first and second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product introduced way by the second group.

24. Find a vector in the direction of vector $2\hat{i} - 3\hat{j} + 6\hat{k}$ which has magnitude of 21 units.

OR

Find the angle between X-axis and the vector $\hat{i} + \hat{j} + \hat{k}$.

25. Evaluate $\int \cos^{-1}(\sin x)\, dx$.

OR

Write the anti-derivative of $\left(3\sqrt{x} + \frac{1}{\sqrt{x}}\right)$.

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Find the area of a parallelogram whose adjacent sides represented by the vectors $2\hat{i} - 3\hat{k}$ and $4\hat{i} + 2\hat{k}$.

27. Using the principal values, write the value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

28. If $y = \sin^{-1}(6x\sqrt{1 - 9x^2})$, $-\frac{1}{3\sqrt{2}} < x < \frac{1}{3\sqrt{2}}$, then find $\frac{dy}{dx}$.

OR

If $(\cos x)^y = (\cos y)^x$, then find $\frac{dy}{dx}$.

29. If $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$, write the value of x.

30. The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2 cm.

OR

Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is always increasing on R.

31. Evaluate $\int \frac{2\cos x}{\sin^2 x} dx$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Solve the differential equation $\frac{dy}{dx} - 3y\cot x = \sin 2x$, given $y = 2$ when $x = \frac{\pi}{2}$.

33. Maximize $Z = 3x + 2y$ subject to $x + 2y \leq 0$, $3x + y \leq 15$, x, $y \geq 0$.

OR

Minimise $Z = x + 2y$ subject to $2x + y \geq 3$, $x + 2y \geq 6$, x, $y \geq 0$. Show that the minimum of Z occurs at more than two points.

34. Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also, find their point intersection.

35. Find $\int \frac{x^2 + x + 1}{(x+2)(x^2+1)} dx$.

OR

Evaluate $\int_0^{\pi} e^{2x} \cdot \sin\left(\frac{\pi}{4} + x\right) dx$.

Section - E

Case study based questions are compulsory.

36. Federal health officials have reported that the proportion of children (ages 19 to 35 months) who received a full series of inoculations against vaccine-preventable diseases, including diphtheria, tetanus, measles, and mumps, increased up until 2006, but has stalled since. The CDC reports that 14 states have achieved a vaccination coverage rate of at least 80% for the 4:3:1:3:3:1 series.26 The probability that a randomly selected toddler in Alabama has received a full set of inoculations is 0.792, for a toddler in Georgia, 0.839, and for a toddler in Utah, 0.711.27 Suppose a toddler from each state is randomly selected.

(i) Find the probability that all three toddlers have received these inoculations.
(ii) Find the probability that none of the three has received these inoculations.

37. Cross holding, also referred to as cross shareholding, describes a situation where one publicly-traded company holds a significant number of shares of another publicly-traded company. The shares owned of the second publicly-traded company are referred to as a cross-holding of the first company.

Two companies A and B are holding shares in each other. A is holding 20% shares of B and B is holding 10% shares. of A. The separately earned profits of the two companies are ₹ 98000 and ₹ 49000 respectively.

(i) Find total profit of each company using matrix notations.

(ii) Show that the total of the profits allocated to outside shareholders is equal to the total of separately earned profit.

38. Ravindra Manch was established in 1963 to commemorate the 100th birth anniversary of Ravindra Nath Tagore. Ravindra Manch is one of the myriad places in Jaipur that hold a historical significance. The auditorium was among the seventeen cultural centers that were envisioned by Pandit Jawaharlal Nehru and was thrown open to the public on Independence Day in the year 1963. Since then, the place has hosted a wide number of cultural shows and events. Some of the most renowned artists, dancers and actors have displayed their talent at this prestigious venue.

Last year, 300 people attended the Ravindra Manch Drama Club's winter play. The ticket price was < 70. The advisor estimates that 20 fewer people would attend for each < 10 increase in ticket price.

(i) What ticket price would give the most income for the Drama Club?

(ii) If the Drama Club raised its tickets to this price, how much income should it expect to bring in?

□□□□□□□

Sample Paper 08

Class - 12th Exam - 2024 - 25

Mathematics (Code-041)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided some questions.
9. Use of calculators is not allowed.

Section - A

Section A consists of 20 questions of 1 mark each.

1. The function $f(x) = x^3$ has a

(a) local minima at $x = 0$

(b) local maxima at $x = 0$

(c) point of inflexion at $x = 0$

(d) none of the above

2. Find the area of a curve $xy = 4$, bounded by the lines $x = 1$ and $x = 3$ and X-axis.

(a) $\log 12$

(b) $\log 64$

(c) $\log 81$

(d) $\log 27$

3. Two vectors $\vec{a}$ and $\vec{b}$ are parallel and have same magnitude, then

(a) they have the same direction

(b) they are equal

(c) they are not equal

(d) they may or may not be equal

4. If $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$ are two sets and function $f : A \to B$ is defined by $f(x) = x + 2$, $\forall x \in A$, then the function f is

(a) bijective

(b) onto

(c) one-one

(d) many-one

5. The function $f(x) = x^2$, for all real x, is

(a) decreasing

(b) increasing

(c) neither decreasing nor increasing

(d) none of the above

6. $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ is equal to

(a) 0
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$
(d) π

7. If the position vectors of the vertices A, B, C of a triangle ABC are $7\hat{j} + 10\hat{k}, -\hat{i} + 6\hat{j} + 6\hat{k}$ and $-4\hat{i} + 9\hat{j} + 6\hat{k}$ respectively, then triangle is

(a) equilateral
(b) isosceles
(c) scalene
(d) right angled and isosceles also

8. The order of the differential equation of all circles of radius a is

(a) 2
(b) 3
(c) 4
(d) 1

9. The derivative of $\log|x|$ is

(a) $\frac{1}{x}, x > 0$
(b) $\frac{1}{|x|}, x \neq 0$
(c) $\frac{1}{x}, x \neq 0$
(d) None of these

10. $\cos^{-1}\left[\cos\left(-\frac{\pi}{3}\right)\right]$ is equal to

(a) $-\frac{\pi}{3}$
(b) $\frac{\pi}{3}$
(c) $\frac{2\pi}{3}$
(d) $\frac{5\pi}{3}$

11. If $A = \begin{bmatrix} 1 & x \\ x^2 & 4y \end{bmatrix}$, $B = \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix}$ and $adj\,(A) + B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the values of x and y are

(a) 1, 1
(b) ± 1, 1
(c) 1, 0
(d) None of these

12. If $f(x) = \begin{cases} ax + 3, & x \leq 2 \\ a^2x - 1, & x > 2 \end{cases}$, then the values of a for which f is continuous for all x are

(a) 1 and -2
(b) 1 and 2
(c) -1 and 2
(d) -1 and -2

13. Solution of $(x + 2y^3)\,dy = y\,dx$ is

(a) $x = y^3 + cy$
(b) $x + y^3 = cy$
(c) $y^2 - x = cy$
(d) none of these

14. Two dice are thrown n times in succession. The probability of obtaining a doublet six atleast once is

(a) $\left(\frac{1}{36}\right)^n$
(b) $1 - \left(\frac{35}{36}\right)^n$
(c) $\left(\frac{1}{12}\right)^n$
(d) None of these

15. Integrating factor of differential equation $\cos x \frac{dy}{dx} + y \sin x = 1$

(a) $\cos x$ (b) $\tan x$

(c) $\sec x$ (d) $\sin x$

16. If α, β, γ are the angles which a half ray makes with the positive directions of the axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is equal to

(a) 2 (b) 1

(c) 0 (d) -1

17. $\int \frac{1 + x + \sqrt{x + x^2}}{\sqrt{x} + \sqrt{1 + x}} dx$ is equal to

(a) $\frac{1}{2}\sqrt{1 + x} + C$ (b) $\frac{2}{3}(1 + x)^{3/2} + C$

(c) $\sqrt{1 + x} + C$ (d) $2(1 + x)^{3/2} + C$

18. If $P(A \cup B) = 0.83$, $P(A) = 0.3$ and $P(B) = 0.6$, then the events will be

(a) dependent (b) independent

(c) cannot say anything (d) None of the above

19. **Assertion:** If $A = \frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}$, then $(A^T)A = I$

Reason: For any square matrix A, $(A^T)^T = A$

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

20. **Assertion:** Let $A = \{-1, 1, 2, 3\}$ and $B = \{1, 4, 9\}$ where $f: A \rightarrow B$ given by $f(x) = x^2$, then f is a many-one function.

Reason: If $x_1 \neq x^2 \Rightarrow f(x_1) \neq f(x_2)$, for every $x_1, x_2 \in$ domain then f is one-one or else many

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Find the vector equation of the line which passes through the point $(3, 4, 5)$ and is parallel to the vector $2\hat{i} + 2\hat{j} - 3\hat{k}$.

22. Prove that if E and F are independent events, then the events E and F' are also independent.

23. Find $\int \frac{\sin^6 x}{\cos^8 x} dx$.

OR

Evaluate $\int \frac{dx}{\sin^2 x \cos^2 x}$.

24. Write the direction ratios of the vector $3\vec{a} + 2\vec{b}$, where $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$.

OR

Find the unit vector in the direction of the sum of vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $4\hat{i} - 3\hat{j} + 2\hat{k}$.

25. Let R is the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$. Write the equivalence class $[0]$.

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Write a unit vector perpendicular to both the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$.

27. Write the value of $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$.

28. Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$

29. Evaluate $\int \frac{2\cos x}{3\sin^2 x} dx$.

30. Find the value of k, so that the following functions is continuous at $x = 2$. $f(x) = \begin{cases} \frac{x^3 - x^2 - 16x + 20}{(x-2)^2}, & x \neq 2 \\ k, & x = 2 \end{cases}$

OR

Find $\frac{dy}{dx}$ *at* $x = 1$, $y = \frac{\pi}{4}$ if $\sin^2 y + \cos xy = K$.

31. The volume of a sphere is increasing at the rate of 8 cm3/s. Find the rate of which its surface area is increasing when the radius of the sphere is 12 cm.

OR

Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Show that the differential equation $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + x\,dy = 0$ is homogeneous. Find the particular solution of this differential equation, given that $y = \frac{\pi}{4}$, when $x = 1$.

33. Maximise $Z = 5x + 3y$ subject to the constraints: $3x + 5y \leq 15$; $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

OR

Minimize $Z = 3x + 5y$ such that $x + 3y \geq 3$, $x + y \geq 2$, x, $y \geq 0$.

34. Show that the line lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect. Also, find their point of intersection.

35. Find the value of $\int \frac{\cos x}{(1 + \sin x)(2 + \sin x)}dx$.

OR

Prove that $\int_0^a f(x)\,dx = \int_0^a f(a - x)\,dx$. and hence evaluate $\int_0^{\pi/2} \frac{x}{\sin x + \cos x}dx$.

Section - E

Case study based questions are compulsory.

36. In apparels industries retailers have an interesting conundrum facing them. On one hand, consumers are more drawn to hot promotional deals than ever before. The result of this is that they sell more units (of product) for less money, and this adversely impacts comp store sales.

Arvind Fashions knows that the it can sell 1000 shirts when the price is ₹ 400 per shirt and it can sell 1500 shirts when the price is ₹ 200 a shirt. Determine

(i) the price function

(ii) the revenue function

(iii) the marginal revenue function.

37. A market analysis is a quantitative and qualitative assessment of a market. It looks into the size of the market both in volume and in value, the various customer segments and buying patterns, the competition, and the economic environment in terms of barriers to entry and regulation.

Based on the past marketing trends and his own experience, marketing expert suggested to the concerned the segments of market for their products as follows:
The first segment consisted of lower income class, the second segment that of middle income and the third segment that of high income. The data based on the income of the consumers was readily available. During a particular month in particular year, the agent reported that for three products of the company the following were the sales: There were 200 customers who bought all the three products, 240 customers who bought I and III, 60 customers only products II and II and 80 customers only products only III regardless of the market segmentation groups.
Based on the market segmentation analysis, for product I, the percentage for the income groups are given as (40%, 20% and 40%), for product II (30%, 20% and 50%), for product III (10%, 50% and 40%).

(i) Taking the suitable variable form the system of equation that represent given problem.
(ii) Using matrix method, find out the number of persons in the lower income, middle income and higher income class in the region referred.

38. The U.S. Constitution directs the government to conduct a census of the population every 10 years. Population totals are used to allocate congressional seats, electoral votes, and funding for many government programs. The U.S. Census Bureau also compiles information related to income and poverty, living arrangements for children, and marital status. The following joint probability table lists the probabilities corresponding to marital status and sex of persons 18 years and over.14

Sex	**Marital Status**			
	(R)	(N)	(W)	(D)
(M)	0.282	0.147	0.013	0.043
(F)	0.284	0.121	0.050	0.060

$R \Rightarrow$ Married
$N \Rightarrow$ Never married
$W \Rightarrow$ Widowed
$D \Rightarrow$ Discovered or separated
$M \Rightarrow$ Male
$F \Rightarrow$ Male

Suppose a U.S. resident 18 years or older is selected at random.

(i) Find the probability that the person is female and widowed.
(ii) Suppose the person is male. What is the probability that he was never married?
(iii) Suppose the person is married. What is the probability that the person is female?

□□□□□□□

Sample Paper 09

Class - 12th Exam - 2024 - 25

Mathematics (Code-041)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided some questions.
9. Use of calculators is not allowed.

Section - A

Section A consists of 20 questions of 1 mark each.

1. The projection of $\vec{a} = 3\hat{i} - \hat{j} + 5\hat{k}$ on $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$ is

(a) $\frac{8}{\sqrt{35}}$ (b) $\frac{8}{\sqrt{39}}$

(c) $\frac{8}{\sqrt{14}}$ (d) $\sqrt{14}$

2. If $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and A^2 is the identity matrix, then x is equal to

(a) -1 (b) 0

(c) 1 (d) 2

3. If $x = \frac{2\,at}{1+t^3}$ and $y = \frac{2\,at^2}{(1+t^3)^2}$, then $\frac{dy}{dx}$ is equal to

(a) ax (b) a^2x^2

(c) $\frac{x}{a}$ (d) $\frac{x}{2a}$

4. A sphere increases its volume at the rate of π cm3/s. The rate at which its surface area increases, when the radius is 1 cm is

(a) 2π sq cm/s (b) π sq cm/s

(c) $\frac{3\pi}{2}$ sq cm/s (d) $\frac{\pi}{2}$ sq cm/s

5. $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ is equal to

(a) 0 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

6. $3a \int_0^1 \left(\frac{ax - 1}{a - 1}\right)^2 dx$ is equal to

(a) $a - 1 + (a - 1)^{-2}$ (b) $a + a^{-2}$

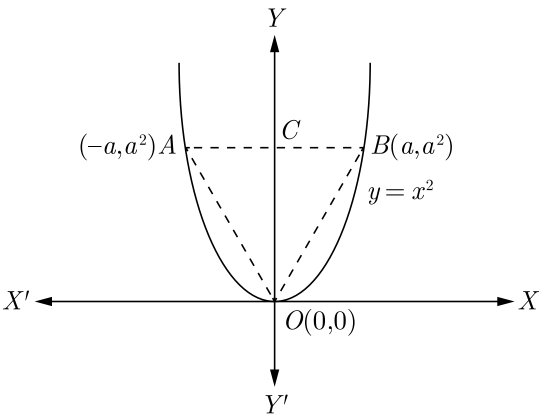
(c) $a - a^2$ (d) $a^2 + \frac{1}{a^2}$

7. If $\int_0^a f(2a - x)\, dx = m$ and $\int_0^a f(x)\, dx = n$, then $\int_0^{2a} f(x)\, dx$ is equal to

(a) $2m + n$ (b) $m + 2n$

(c) $m - n$ (d) $m + n$

8. The given figure shows a $\Delta\, AOB$ and the parabola $y = x^2$. The ratio of the area of the $\Delta\, AOB$ to the area of the region AOB of the parabola $y = x^2$ is equal to



(a) $\frac{3}{5}$ (b) $\frac{3}{4}$

(c) $\frac{7}{8}$ (d) $\frac{5}{6}$

9. The angle between the lines $x = 1,\ y = 2$ and $y = -1,\ z = 0$ is

(a) 30° (b) 60°

(c) 90° (d) 0°

10. Order of the equation $\left(1 + 5\frac{dy}{dx}\right)^{3/2} = 10\frac{d^3y}{dx^3}$ is

(a) 2 (b) 3

(c) 1 (d) 0

11. If x is real, then minimum value of $x^2 - 8x + 17$ is

(a) -1 (b) 0

(c) 1 (d) 2

12. $y = 2e^{2x} - e^{-x}$ is a solution of the differential equation

(a) $y_2 + y_1 + 2y = 0$ (b) $y_2 - y_1 + 2y = 0$

(c) $y_2 + y_1 = 0$ (d) $y_2 - y_1 - 2y = 0$

13. If $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.3$, then $P\left(\frac{A'}{B}\right)$ is equal to

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

14. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$. Then, which one of the following is correct?

(a) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} = 0$

(b) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq 0$

(c) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{a} \times \vec{c} = 0$

(d) $\vec{a} \times \vec{b}$, $\vec{b} \times \vec{c}$, $\vec{c} \times \vec{a}$ are mutually perpendicular.

15. The line joining the points $(1,1,2)$ and $(3,-2,1)$ meets the planes $3x + 2y + z = 6$ at the point

(a) $(1,1,2)$ (b) $(3,-2,1)$

(c) $(2,-3,1)$ (d) $(3,2,1)$

16. If $f(x) = \begin{cases} \frac{3\sin \pi x}{5x}, & x \neq 0 \\ 2k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

(a) $\frac{\pi}{10}$ (b) $\frac{3\pi}{10}$

(c) $\frac{3\pi}{2}$ (d) $\frac{3\pi}{5}$

17. For two events A and B, if $P(A) = P\left(\frac{A}{B}\right) = \frac{1}{4}$ and $P\left(\frac{B}{A}\right) = \frac{1}{2}$, then

(a) A and B are independent events (b) $P\left(\frac{A'}{B}\right) = \frac{3}{4}$

(c) $P\left(\frac{B'}{A}\right) = \frac{1}{2}$ (d) All of the above

18. The area bounded by $y = |\sin x|$, X-axis and the lines $|x| = \pi$ is

(a) 2 sq units (b) 3 sq units

(c) 4 sq units (d) None of these

19. **Assertion :** If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then $\text{adj}(\text{adj}\,A) = A$.

Reason : $|\text{adj}(\text{adj}\,A)| = |A|^{(n-1)^2}$, where A be n rowed non-singular matrix.

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

20. **Assertion :** The matrix $A = \frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}$ is an orthogonel matrix.

Reason : If A and B are orthagonal, then AB is also orthegonal.

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Find the vector equation of the line passing through the point $A(1, 2, -1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$.

OR

The x-coordinate of point on the line joining the points $P(2, 2, 1)$ and $Q(5, 1, -2)$ is 4. Find its z -coordinate.

22. Show that the function $f(x) = x^3 - 3x^2 + 3x,\ x \in R$ is increasing on R.

23. Find the general solution of following equation $e^x \tan y\, dx + (1 - e^x)\sec^2 y\, dy = 0$

24. If $P(not\ A) = 0.7$, $P(B) = 0.7$ and $P(B/A) = 0.5$, then find $P(A/B)$.

25. Maximize $Z = x + y$,
subject to $x - y \le -1$, $x + y \le 0$, x, $y \ge 0$.

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Show that $y = \log(1 + x) - \frac{2x}{2 + x}$, $x > -1$ is an increasing function of x throughout its domain.

27. Show that the modulus function $f: R \to R$, given by $f(x) = [x]$, is neither one-one nor onto, where $|x|$ is x, if x is positive or 0 and $|x|$ is $-x$, if x is negative.

OR

Show that the Signum function of $f: R \to R$, given by $f(x) = \begin{cases} 1, & if \ \ x > 0 \\ 0, & if \ \ x = 0 \\ -1, & if \ \ x < 0 \end{cases}$ is neither one-one nor onto.

28. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

29. Maximise and minimise $Z = x + 2y$ subject to the constraints

$x + 2y \geq 100, \ 2x - y \leq 0, \ 2x + y \leq 200, \ x, \ y \geq 0$

Solve the above LPP graphically.

30. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

31. Find $\int \frac{(2x-5)\,e^{2x}}{(2x-3)^3}\,dx$.

OR

Evaluate $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\,dx$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Using integration, find the area of the region bounded by the triangle whose vertices are $(-1,0)$, $(1,3)$ and $(3,2)$.

OR

Using integration, find the area of the triangular region whose have the equation $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

33. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right)$, $x^2 \leq 1$, then find dy/dx.

OR

Find whether the following function is differentiable at $x = 1$ and $x = 2$ or not.

$$f(x) = \begin{cases} x, & x < 1 \\ 2 - x, & 1 \leq x \leq 2 \\ -2 + 3x - x^2, & x > 2 \end{cases}$$

34. Using vectors, find the area of the ΔABC, whose vertices are $A(1,2,3)$, $B(2,-1,4)$ and $C(4,5,-1)$.

OR

If lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$

intersect, then find the value of k and hence, find the equation of the plane containing these lines.

35. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b)R(c, d)$ if $ad(b+c) = bc(a+d)$. Show that R is an equivalence relation.

OR

Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto.

Section - E

Case study based questions are compulsory.

36. Brine is a high-concentration solution of salt in water. In diverse contexts, brine may refer to the salt solutions ranging from about 3.5% up to about 26%. Brine forms naturally due to evaporation of ground saline water but it is also generated in the mining of sodium chloride.

A tank initially contains 10 gallons of pure water. Brine containing 3 pounds of salt per gallon flows into the tank at a rate of 2 gallons per minute, and the well-stirred mixture flows out of the tank at the same rate.

(i) How much salt is present at the end of 10 minutes?

(ii) How much salt is present in the long run?

37. ICAR-Indian Agricultural Research Institute is an autonomous body responsible for co-ordinating agricultural education and research in India. It reports to the Department of Agricultural Research and Education, Ministry of Agriculture. The Union Minister of Agriculture serves as its president. It is the largest network of agricultural research and education institutes in the world.

ICAR grows vegetables and grades each one as either good or bad for its taste, good or bad for its size, and good or bad for its appearance. Overall 78% of the vegetables have a good taste. However, only 69% of the vegetables have both a good taste and a good size. Also, 5% of the vegetables have both a good taste and a good appearance, but a bad size. Finally, 84% of the vegetables have either a good size or a good appearance.

(i) If a vegetable has a good taste, what is the probability that it also has a good size?

(ii) If a vegetable has a bad size and a bad appearance, what is the probability that it has a good taste?

38. Pastry is a dough of flour, water and shortening that may be savoury or sweetened. Sweetened pastries are often described as bakers' confectionery. The word "pastries" suggests many kinds of baked products made from ingredients such as flour, sugar, milk, butter, shortening, baking powder, and eggs.

The Sunrise Bakery Pvt Ltd produces three basic pastry mixes A, B and C. In the past the mix of ingredients has shown in the following matrix :

$$\begin{array}{cc} & \begin{array}{ccc} \text{Flour} & \text{Fat} & \text{Sugar} \end{array} \\ \text{Type} \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 5 & 1 & 1 \\ 6.5 & 2.5 & 0.5 \\ 4.5 & 3 & 2 \end{bmatrix} \end{array} \text{(All quantities in kg)}$$

Due to changes in the consumer's tastes it has been decided to change the mixes using the following amendment matrix :

$$\begin{array}{cc} & \begin{array}{ccc} \text{Flour} & \text{Fat} & \text{Sugar} \end{array} \\ \text{Type} \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ -0.5 & 0.5 & 0.5 \\ 0.5 & 0 & 0 \end{bmatrix} \end{array}$$

Using matrix algebra you are required to calculate :

(i) the matrix for the new mix:

(ii) the production requirement to meet an order for 50 units of type A, 30 units of type B and 20 units of type C of the new mix;

(iii) the amount of each type that must be made to totally use up 370 kg of flour, 170 kg of fat and 80 kg of sugar that are at present in the stores.

□□□□□□□□

Sample Paper 10

Class - 12th Exam - 2024 - 25

Mathematics (Code-041)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided some questions.
9. Use of calculators is not allowed.

Section - A

Section A consists of 20 questions of 1 mark each.

1. If α, β, γ are the angles which a half ray makes with the positive directions of the axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is equal to

(a) 2

(b) 1

(c) 0

(d) -1

2. Which of the following triplets gives the direction cosines of a line?

(a) $< 1, 1, 1 >$

(b) $< 1, -1, 1 >$

(c) $< 1, -1, -1 >$

(d) $< \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$

3. Solution of $\frac{dy}{dx} + y \sec x = \tan x$ is

(a) $y(\sec x + \tan x) = \sec x + \tan x - x + c$

(b) $y = \sec x + \tan x - x + c$

(c) $y(\sec x + \tan x) = \sec x + \tan x + x + c$

(d) none of the above

4. The function $f(x) = 2 - 3x$ is

(a) decreasing

(b) increasing

(c) neither decreasing nor increasing

(d) none of the above

5. If $y = \log[\sin(x^2)]$, $0 < x < \frac{\pi}{2}$, then $\frac{dy}{dx}$ at $x = \frac{\sqrt{\pi}}{2}$ is

(a) 0

(b) 1

(c) $\pi/4$

(d) $\sqrt{\pi}$

6. The radius of a cylinder is increasing at the rate of 3m/s and its altitude is decreasing at the rate of 4m/s . The rate of change of volume when radius is 4m and altitude is 6m, is

(a) $80\pi m^3/s$ (b) $144\pi m^3/s$

(c) $80 m^3/s$ (d) $64 m^3/s$

7. $\int \sqrt{1+\cos x}\, dx$ is equal to

(a) $2\sin\left(\frac{x}{2}\right)+C$ (b) $\sqrt{2}\sin\left(\frac{x}{2}\right)+C$

(c) $2\sqrt{2}\sin\left(\frac{x}{2}\right)+C$ (d) $\frac{1}{2}\sin\left(\frac{x}{2}\right)+C$

8. The value of $\int_{-2}^{2}(x\cos x+\sin x+1)\,dx$ is

(a) 2 (b) 0

(c) -2 (d) 4

9. If A is 3×4 matrix and B is a matrix such that $A'B$ and BA' are both defined, then B is of the type

(a) 4×4 (b) 3×4

(c) 4×3 (d) 3×3

10. If A is a matrix of order 3 such that $A(adj\ A)=10I$, then $|adj\ A|$ is equal to

(a) 1 (b) 10

(c) 100 (d) $10I$

11. If $|x|\le 1$, which of the following four is different from the other three?

(a) $\sin(\cos^{-1}x)$ (b) $\cos(\sin^{-1}x)$

(c) $\sqrt{1-x^2}$ (d) $\frac{\sqrt{1-x^2}}{x}$

12. The probability distribution of a random variable X is given below

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
$P(X)$	p	$2p$	$3p$	$4p$	$5p$	$7p$	$8p$	$9p$	$10p$	$11p$	$12p$

Then, the value of p is

(a) $\frac{1}{72}$ (b) $\frac{3}{73}$

(c) $\frac{5}{72}$ (d) $\frac{1}{74}$

13. If $f(x)=\begin{cases}\frac{3\sin\pi x}{5x}, & x\neq 0\\ 2k, & x=0\end{cases}$ is continuous at $x=0$, then the value of k is

(a) $\frac{\pi}{10}$ (b) $\frac{3\pi}{10}$

(c) $\frac{3\pi}{2}$ (d) $\frac{3\pi}{5}$

14. The area of the region bounded by the lines $y=mx$, $x=1$, $x=2$ and X-axis is 6 sq units, then m is equal to

(a) 3 (b) 1

(c) 2 (d) 4

15. The function $f(x) = x^2 + bx + c$, where b and c are real constants, describes

(a) one-one mapping

(b) onto mapping

(c) not one-one but onto mapping

(d) neither one-one nor onto mapping

16. The equation of the curve, whose slope at any point different from origin is $y + \frac{y}{x}$, is

(a) $y = cxe^x, c \neq 0$

(b) $y = xe^x$

(c) $xy = e^x$

(d) $y + xe^x = c$

17. The differential equation representing the family of curve $y^2 = (x + \sqrt{c})$, where c is positive perimeter, is of

(a) order 1, degree 3

(b) order 2, degree 2

(c) degree 3, order 1

(d) degree 4, order 4

18. Let $\vec{a}$ and $\vec{b}$ be two non-parallel unit vectors in a plane. If the vectors $(\alpha\vec{a} + \vec{b})$ bisects the internal angle between $\vec{a}$ and $\vec{b}$, then α is equal to

(a) 1

(b) 1/2

(c) 4

(d) 2

19. **Assertion:** If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew symmetric matrix, then value of $(a - b - c)$ is 1

Reason: A square matrix $A = [a_{ij}]$ is said to be skew-symmetric if $A' = -A$

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

20. **Assertion:** Two dice are tossed the following two events A and B are $A = \{(x, y): x + y = 11\}$, $B = \{(x, y): x \neq 5\}$ independent events.

Reason: E_1 and E_2 are independent events, then $P(E_1 \cap E_2) = P(E_1)P(E_2)$

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. If a line makes angles 90°, 135°, 45° with then x, y and z axes respectively, find the direction consines.

22. If $P(not\ A) = 0.7$, $P(B) = 0.7$ and $P(B/A) = 0.5$, then find $P(A/B)$.

23. Find $\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} dx$.

OR

Find $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$.

24. Find the position vector of a point which divides the join of points with position vectors $\vec{a} - 2\vec{b}$ and $2\vec{a} + \vec{b}$ externally in the ratio 2 : 1.

OR

If $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, then find a unit vector parallel to the vector $\vec{a} + \vec{b}$.

25. If $R = \{(a, a^3)$: a is a prime number less than 5$\}$ be a relation. Find the range of R.

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. The total cost $C(x)$ associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$.
Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

OR

The total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$ in rupees. Find the marginal revenue we mean the rate of change of total revenue with respect to the number of times sold at an instant.

27. Evaluate $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$.

28. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then find the value of $|2\hat{a} + \hat{b} + \hat{c}|$.

29. Determine the value of k for which the following function is continuous at $x = 3$:

$$f(x) = \begin{cases} \frac{(x+3)^2 - 36}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$

OR

Determine the value of constant k so that the function $f(x) = \begin{cases} \frac{kx}{|x|}, & if\ x < 0 \\ 3, & if\ x \geq 0 \end{cases}$ is continuous at $x = 0$.

30. Find $|AB|$, if $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$.

31. Write the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$ given that $y = 1$, when $x = 0$.

33. Maximize $Z = 3x + 4y$, subject to the constraints; $x + y \leq 4$, $x \geq 0$, $y \geq 0$.

OR

Minimize $Z = -3x + 4y$ subject to the constraints $x + 2y \leq 8$, $3x + 2y \leq 12$, $x \geq 0$, $y \geq 0$

34. Find : $\int \frac{3x + 5}{x^2 + 3x - 18} dx$.

OR

Prove that $\int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx$, hence evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$.

35. Prove that the line through $A(0, -1, -1)$ and $B(4, 5, 1)$ intersects the line through $C(3, 9, 4)$ and $D(-4, 4, 4)$.

Section - E

Case study based questions are compulsory.

36. Hindustan Pencils Pvt. Ltd. is an Indian manufacturer of pencils, writing materials and other stationery items, established in 1958 in Bombay. The company makes writing implements under the brands Nataraj and Apsara, and claims to be the largest pencil manufacturer in India.

Hindustan Pencils manufactures x units of pencil in a given time, if the cost of raw material is square of the pencils produced, cost of transportation is twice the number of pencils produced and the property tax costs ₹ 5000. Then,

(i) Find the cost function $C(x)$.

(ii) Find the cost of producing 21st pencil.

(iii) The marginal cost of producing 50 pencils.

37. OYO Rooms, also known as OYO Hotels & Homes, is an Indian multinational hospitality chain of leased and franchised hotels, homes and living spaces. Founded in 2012 by Ritesh Agarwal, OYO initially consisted mainly of budget hotels.

Data analyst at OYO say that during frequent trips to a certain city, a traveling salesperson stays at hotel A 50% of the time, at hotel B 30% of the time, and at hotel C 20% of the time. When checking in, there is some problem with the reservation 3% of the time at hotel A, 6% of the time at hotel B, and 10% of the time at hotel C. Suppose the salesperson travels to this city.

(i) Find the probability that the salesperson stays at hotel A and has a problem with the reservation.

(ii) Find the probability that the salesperson has a problem with the reservation.

(iii) Suppose the salesperson has a problem with the reservation; what is the probability that the salesperson is staying at hotel A?

38. A car carrier trailer, also known as a car-carrying trailer, car hauler, or auto transport trailer, is a type of trailer or semi-trailer designed to efficiently transport passenger vehicles via truck. Commercial-size car carrying trailers are commonly used to ship new cars from the manufacturer to auto dealerships. Modern car carrier trailers can be open or enclosed. Most commercial trailers have built-in ramps for loading and off-loading cars, as well as power hydraulics to raise and lower ramps for stand-alone accessibility.

A transport company uses three types of trucks T_1, T_2 and T_3 to transport three types of vehicles V_1, V_2 and V_3. The capacity of each truck in terms of three types of vehicles is given below:

	V_1	V_2	V_3
T_1	1	3	2
T_2	2	2	3
T_3	3	2	2

Using matrix method find :

(i) The number of trucks of each type required to transport 85, 105 and 110 vehicles of V_1, V_2 and V_3 types respectively.

(ii) Find the number of vehicles of each type which can be transported if company has 10, 20 and 30 trucks of each type respectively.

□□□□□□□

Sample Paper 11

Class - 12th Exam - 2024 - 25

Mathematics (Code-041)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided some questions.
9. Use of calculators is not allowed.

Section - A

Section A consists of 20 questions of 1 mark each.

1. Maximum value of $f(x) = \sin x + \cos x$ is

(a) 1 (b) 2

(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

2. $f(x) = 2x^3 - 15x^2 + 36x + 4$ is

(a) increasing in $(-\infty, 2]$ (b) increasing in [2, 3]

(c) decreasing in $(3, \infty)$ (d) None of these

3. The optimal value of the objective function is attached at the point:

(a) given by intersection of inequations with axes only.

(b) given by intersection of inequations with x-axis only.

(c) given by corner points of the feasible region.

(d) none of the above.

4. If $A = \{1, 2, 3\}$, then how many equivalence relation can be defined on A containing (1, 2) :

(a) 2 (b) 3

(c) 8 (d) 6

5. $\frac{d}{dx}[\log x] = ?$

(a) $\frac{1}{x}$

(c) $-\frac{1}{x^2}$

(c) 1

(d) $\frac{1}{x^2}$

6. $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = ?$

(a) $\frac{\pi}{4}$

(b) $-\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) $-\frac{\pi}{2}$

7. $A = \begin{bmatrix} 3 & 6 \\ 5 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 8 \\ 5 & 6 \end{bmatrix}$, $2A + 3B = ?$

(a) $\begin{bmatrix} 27 & 24 \\ 22 & 10 \end{bmatrix}$

(b) $\begin{bmatrix} 27 & 36 \\ 25 & 10 \end{bmatrix}$

(c) $\begin{bmatrix} 27 & 36 \\ 25 & 15 \end{bmatrix}$

(d) $\begin{bmatrix} 27 & 36 \\ 35 & 10 \end{bmatrix}$

8. $\int 0.dx =$ ....

(a) k

(b) 0

(c) 1

(d) -1

9. $\int_2^1 \frac{dx}{x} = ?$

(a) $\log\frac{2}{3}$

(b) $\log\frac{3}{2}$

(c) $\log\frac{1}{2}$

(d) $\log\frac{x}{2}$

10. Integrating factor (IF) of the differential equation $\frac{dy}{dx} - y\cos x = \sin x\cos x$

(a) $e^{-\sin x}$

(b) $e^{\sin x}$

(c) $e^{-\cos x}$

(d) $e^{\cos x}$

11. The solution of the differential equation $\frac{dy}{dx} = \frac{x}{y}$ is

(a) $x - y = k$

(b) $x^2 - y^2 = k$

(c) $x^3 - y^3 = k$

(d) $xy = k$

12. $\frac{d}{dx}(\sin x) =$

(a) $\cos x$

(b) $-\sin x$

(c) $-\cos x$

(d) $\tan x$

13. $\hat{i} \times (\hat{i} \times \hat{j}) + \hat{j} \times (\hat{j} \times \hat{k}) + \hat{k} \times (\hat{k} \times \hat{i}) =$

(a) $\hat{i} + \hat{j} + \hat{k}$

(b) 0

(c) 1

(d) $-(\hat{i} + \hat{j} + \hat{k})$

14. The real number which most exceeds its cube is

(a) $\frac{1}{\sqrt{3}}$

(b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}$

(d) None of these

15. If the direction cosines of two straight lines are l_1, m_1, n_1 and l_2, m_2, n_2 then the cosine of the angle θ between them or $\cos\theta$ is

(a) $(l_1 + m_1 + n_1)(l_2 + m_2 + n_2)$

(b) $\frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2}$

(c) $l_1 l_2 + m_1 m_2 + n_1 n_2$

(d) $\frac{l_1 + m_1 + n_1}{l_2 + m_2 + n_2}$

16. If $\lambda \in R$ and $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ then $\lambda\Delta =$

(a) $\begin{vmatrix} \lambda a & \lambda b \\ \lambda c & \lambda d \end{vmatrix}$

(b) $\begin{vmatrix} \lambda a & b \\ c & d \end{vmatrix}$

(c) $\begin{vmatrix} \lambda a & b \\ \lambda c & d \end{vmatrix}$

(d) None of these

17. The direction ratios of a straight line are 1,3,5. Its direction cosines are

(a) $\frac{1}{\sqrt{35}}, \frac{3}{\sqrt{35}}, \frac{5}{\sqrt{35}}$

(b) $\frac{1}{9}, \frac{1}{3}, \frac{5}{9}$

(c) $\frac{3}{\sqrt{35}}, \frac{5}{\sqrt{35}}, \frac{1}{\sqrt{35}}$

(d) $\frac{5}{\sqrt{35}}, \frac{3}{\sqrt{35}}, \frac{1}{\sqrt{35}}$

18. $\int \frac{xe^x}{(x+1)^2} dx =$

(a) $\frac{e^x}{(x+1)^2} + c$

(b) $\frac{-e^x}{x+1} + c$

(c) $\frac{e^x}{x+1} + c$

(d) $\frac{-e^x}{(x+1)^2} + c$

19. **Assertion :** $f(x)$ is defined as $f(x) = \begin{cases} x^3 - 3, x \leq 2 \\ x^2 + 1, x > 2 \end{cases}$ is continuous at $x = 2$.

Reason : $f(2) = \lim_{x \to 2} f(x)$.

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true, but reason is not the correct explanation of assertion.

(c) Assertion is true, but reason is false.

(d) Assertion is false, but reason is true.

20. **Assertion** : If A and B are two independent events $P(A \cup B) = 1 - P(A')P(B')$

Reason : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and $P(A \cap B) = P(A)P(B)$

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true, but reason is not the correct explanation of assertion.

(c) Assertion is true, but reason is false.

(d) Assertion is false, but reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Prove, by Vector method, that the angle inscribed in a semi-circle is a right angle.

22. If $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \text{to } \infty}}}$ then $\frac{dy}{dx}$

OR

If $y = \tan(\sin^{-1}x)$ then find $\frac{dy}{dx}$

23. Prove by direction numbers, that the point $(1, -1, 3)$, $(2, -4, 5)$ and $(5, -13, 11)$ are in a straight line.

24. Odds are 8 : 5 against a man, who is 55 years old, living till he is 75 and 4 : 3 against his wife who is now 48, living till she is 68. Find the probability that the couple will be alive 20 years hence.

25. Show that the function $y = ae^{\tan^{-1}x}$ is a solution of the differential equation $(1 + x^2)\frac{d^2y}{dx^2} + (2x - 1)\frac{dy}{dx} = 0$

OR

Show that the function $y = ax + \frac{b}{a}$ is a solution of the differential equation $y = x\frac{dy}{dx} + \frac{b}{\frac{dy}{dx}}$

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. If $\vec{a} = \hat{i} - 2\hat{j} - 3\hat{k}$; $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + 3\hat{j} - 2\hat{k}$ then find $\vec{a} \times (\vec{b} \times \vec{c})$.

27. Thus, $\vec{a} \times (\vec{b} \times \vec{c}) = -(\hat{i} + 8\hat{j} - 5\hat{k})$ If $f: R \to R$ is defined by $f(x) = x^2 - 3x + 2$, find $f\{f(x)\}$.

28. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then find the value of $A^2 + 3A + 2I$.

OR

Find the values of X and $Y : X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.

29. If $y = \log\ \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, show that $\frac{dy}{dx} - \sec x = 0$.

30. The volume of a cube is increasing at a rate of 9 cubic centimetres per second. How fast is the surface area increasing when the length of an edge is 10 cm long?

31. Prove that: $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$

OR

Prove that $4(\cot^{-1}3 + \operatorname{cosec}^{-1}\sqrt{5}) = \pi$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. A random variable has the following probability distribution :

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

(i) k (ii) $P(X < 3)$
(iii) $P(X > 6)$ (iv) $P(0 < x < 3)$

33. Evaluate $\int \frac{5x + 11}{\sqrt{9x^2 + 25}}\,dx$

OR

Prove that : $\int_0^{\pi/2} \log(\tan\theta + \cot\theta)\,d\theta = \pi\log 2$

34. Solve the following L.P.P. graphically:
Minimise and Maximise $Z = 3x + 5y$
Subject to the constraints : $3x - 4y + 12 \geq 0,\ 2x - y + 2 \geq 0, 2x + 3y - 12 \geq 0,\ x \leq 4,\ y \geq 2,\ x \geq 0$

35. Solve the differential equation : $(x - y)\,dy - (x + y)\,dx = 0$

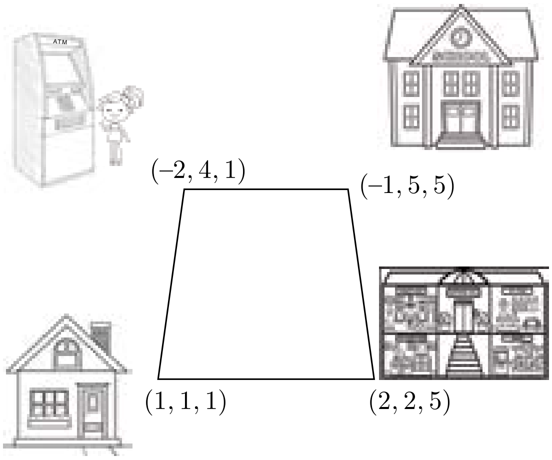
OR

Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$.

Section - E

Case study based questions are compulsory.

36. Lavanya starts walking from his house to shopping mall. Instead of going to the mall directly, she first goes to ATM, from there to her daughter's school and then reaches the mall. In the diagram, using co-ordinate geometry the location of each place is given.



Based on the above information, answer the following questions.

(i) What is the distance between House and ATM?

(ii) What is the distance between ATM and school?

(iii) What is the total distance travelled by Lavanya?

OR

(iv) What is the extra distance travelled by Lavanya in reaching the shopping mall?

37. Pastry is a dough of flour, water and shortening that may be savoury or sweetened. Sweetened pastries are often described as bakers' confectionery. The word "pastries" suggests many kinds of baked products made from ingredients such as flour, sugar, milk, butter, shortening, baking powder, and eggs.

The Sunrise Bakery Pvt Ltd produces three basic pastry mixes A, B and C. In the past the mix of ingredients has shown in the following matrix:

$$\begin{array}{cc} & \begin{array}{ccc}\text{Flour} & \text{Fat} & \text{Sugar}\end{array} \\ \text{Type}\ \begin{array}{c}A\\B\\C\end{array} & \begin{bmatrix} 5 & 1 & 1 \\ 6.5 & 2.5 & 0.5 \\ 4.5 & 3 & 2 \end{bmatrix} \end{array} \text{(All quantities in kg)}$$

Due to changes in the consumer's tastes it has been decided to change the mixes using the following amendment matrix:

$$\begin{array}{cc} & \begin{array}{ccc}\text{Flour} & \text{Fat} & \text{Sugar}\end{array} \\ \text{Type}\ \begin{array}{c}A\\B\\C\end{array} & \begin{bmatrix} 0 & 1 & 0 \\ -0.5 & 0.5 & 0.5 \\ 0.5 & 0 & 0 \end{bmatrix} \end{array}$$

Using matrix algebra you are required to calculate:

(i) the matrix for the new mix:

(ii) the production requirement to meet an order for 50 units of type A, 30 units of type B and 20 units of type C of the new mix;

(iii) the amount of each type that must be made to totally use up 370 kg of flour, 170 kg of fat and 80 kg of sugar that are at present in the stores.

38. A steel can, tin can, tin, steel packaging, or can is a container for the distribution or storage of goods, made of thin metal. Many cans require opening by cutting the "end" open; others have removable covers. They can store a broad variety of contents: food, beverages, oil, chemicals, etc.

A tin can manufacturer a cylindrical tin can for a company making sanitizer and disinfector. The tin can is made to hold 3 litres of sanitizer or disinfector.

Based on the above information, answer the following questions.

(i) If r be the radius and h be the height of the cylindrical tin can, find the surface area expressed as a function of r.

(ii) Find the radius that will minimize the cost of the material to manufacture the tin can.

(iii) Find the height that will minimize the cost of the material to manufacture the tin can.

OR

(iv) If the cost of the material used to manufacture the tin can is $100/\text{m}^2$ find the minimum cost. $\sqrt[3]{\frac{1500}{\pi}} \approx 7.8$

□□□□□□□

Sample Paper 12

Class - 12th Exam - 2024 - 25

Mathematics (Code-041)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided some questions.
9. Use of calculators is not allowed.

Section - A

Section A consists of 20 questions of 1 mark each.

1. If a line makes angles 90°, 60° and θ with X, Y and Z-axis respectively, where θ is acute angle, then find θ.

(a) 30^o (b) 60^o

(c) 45^o (d) 90^o

2. If $\int \frac{x+2}{2x^2+6x+5}dx = P\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{2}\int \frac{dx}{2x^2+6x+5}$, then the value of P is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) 2

3. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(\frac{A}{B}) = \frac{1}{2}$ and $P(\frac{B}{A}) = \frac{2}{3}$. Then, $P(B)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

4. The area bounded by the curve $y = \frac{1}{2}x^2$, the X-axis and the ordinate $x = 2$ is

(a) $\frac{1}{3}$ sq unit (b) $\frac{2}{3}$ sq unit

(c) 1 sq unit (d) $\frac{4}{3}$ sq unit

5. If $A = [a_{ij}]_{2\times 2}$, where $a_{ij} = i + j$, then A is equal to

(a) $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

6. Discuss the continuity of the function $f(x) = \sin 2x - 1$ at the point $x = 0$ and $x = \pi$

(a) Continuous at $x = 0,\ \pi$

(b) Discontinuous at $x = 0$ but continuous at $x = \pi$

(c) Continuous at $x = 0$ but discontinuous at $x = \pi$

(d) Discontinuous at $x = 0,\ \pi$

7. If $f(x) = \begin{cases} \dfrac{\log x}{x-1}, & if\ x \neq 1 \\ k, & if\ x = 1 \end{cases}$ is continuous at $x = 1$, then the value of k is

(a) 0 (b) -1

(c) 1 (d) e

8. Which of the following function is decreasing on $(0, \pi/2)$?

(a) $\sin 2x$ (b) $\cos 3x$

(c) $\tan x$ (d) $\cos 2x$

9. For what values of x, function $f(x) = x^4 - 4x^3 + 4x^2 + 40$ is monotonic decreasing?

(a) $0 < x < 1$ (b) $1 < x < 2$

(c) $2 < x < 3$ (d) $4 < x < 5$

10. The solution of $\dfrac{dy}{dx} = \dfrac{ax + g}{by + f}$ represents a circle, when

(a) $a = b$ (b) $a = -b$

(c) $a = -2b$ (d) $a = 2b$

11. If $\int_0^a f(2a - x)\, dx = m$ and $\int_0^a f(x)\, dx = n$, then $\int_0^{2a} f(x)\, dx$ is equal to

(a) $2m + n$ (b) $m + 2n$

(c) $m - n$ (d) $m + n$

12. Find the area of a curve $xy = 4$, bounded by the lines $x = 1$ and $x = 3$ and X-axis.

(a) $\log 12$ (b) $\log 64$

(c) $\log 81$ (d) $\log 27$

13. Order of the equation $\left(1 + 5\dfrac{dy}{dx}\right)^{3/2} = 10\dfrac{d^3y}{dx^3}$ is

(a) 2 (b) 3

(c) 1 (d) 0

14. The figure formed by four points $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+3\hat{j}$, $3\hat{i}-5\hat{j}-2\hat{k}$, $\hat{k}-\hat{j}$ is a

(a) parallelogram (b) rectangle

(c) trapezium (d) square

15. If $P(A \cup B) = 0.83$, $P(A) = 0.3$ and $P(B) = 0.6$, then the events will be

(a) dependent (b) independent

(c) cannot say anything (d) None of the above

16. Find the equation of the line which passes through the point $(1,2,3)$ and is parallel to the vector $3\hat{i}+2\hat{j}-2\hat{k}$.

(a) $(\hat{i}+\hat{j}+3\hat{k})+\lambda(3\hat{i}+2\hat{j}-2\hat{k})$ (b) $(\hat{i}+2\hat{j}+3\hat{k})+\lambda(3\hat{i}+2\hat{j}-2\hat{k})$

(c) $(\hat{i}+2\hat{j}+5\hat{k})+\lambda(3\hat{i}+2\hat{j}-2\hat{k})$ (d) $(\hat{i}+2\hat{j}+3\hat{k})+\lambda(5\hat{i}+2\hat{j}-2\hat{k})$

17. $\int \sqrt{1+\cos x}\, dx$ is equal to

(a) $2\sin\left(\frac{x}{2}\right)+C$ (b) $\sqrt{2}\sin\left(\frac{x}{2}\right)+C$

(c) $2\sqrt{2}\sin\left(\frac{x}{2}\right)+C$ (d) $\frac{1}{2}\sin\left(\frac{x}{2}\right)+C$

18. The family of curves $y = e^{a\sin x}$, where a is an arbitrary constant is represented by the differential equation

(a) $\log y = \tan x\frac{dy}{dx}$ (b) $y\log y = \tan x\frac{dy}{dx}$

(c) $y\log y = \sin\frac{dy}{dx}$ (d) $\log y = \cos x\frac{dy}{dx}$

19. **Assertion:** if $2P(A) = P(B) = \frac{5}{13}$ and $P\left(\frac{A}{B}\right) = \frac{2}{5}$, then $P(A \cup B)$ is $\frac{11}{26}$

Reason: J, F, E_1 and E_2 are two events. then $P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cup E_2)}{P(E_2)}$, $0 < P(E_2) \leq 1$

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

20. **Assertion :** area of the parallelogram whose adjacent sides are $\hat{i}+\hat{j}-\hat{k}$ and $2\hat{i}-j+\hat{k}$ is $3\sqrt{2}$ square units.

Reason : area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $|\vec{a}-\vec{b}|$

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Determine the value of k for which the following function is continuous at $x = 3$: $f(x) = \begin{cases} \dfrac{(x+3)^2 - 36}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$

OR

Determine the value of constant k so that the function $f(x) = \begin{cases} \dfrac{kx}{|x|}, & if\ x < 0 \\ 3, & if\ x \geq 0 \end{cases}$ is continuous at $x = 0$.

22. If A and B are square matrices of the same order 3, such that $|A| = 2$ and $AB = 2I$. Write the values of $|B|$.

23. If $P(not\ A) = 0.7$, $P(B) = 0.7$ and $P(B/A) = 0.5$, then find $P(A/B)$.

24. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a} + \vec{b}| = 13$ and $|\vec{a}| = 5$, then find the value of $|\vec{b}|$.

25. Find the general solution of the following differential equation $(e^x + e^{-x})dy - (e^x - e^{-x})dx = 0$

Section - C

This section comprises of short answer-type questions (SA) of 3 marks each.

26. Find the particular solution of the differential equation $(+ e^{2x})\, dy + (1 + y^2)\, e^x\, dx = 0$, given that $y = 1$, when $x = 0$.

27. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, find A^{-1}.

Hence, solve the system of equations $x + y + z = 6$, $x + 2z = 7$, $3x + y + z = 12$.

OR

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}.

Use it to solve the system of equations $2x - 3y + 5z = 11.3x + 2y - 4z = -5$, $x + y - 2z = -3$.

28. Find the coordinates of the foot of perpendicular drawn from the point $A(-1,8,4)$ to the line joining the points $B(0,-1,3)$ and $C(2,-3,-1)$. Hence, find the image of the point A in the line BC.

29. Evaluate $\int \dfrac{x^3 + 3x + 4}{\sqrt{x}}$

30. Show that the relation S in the set R of real numbers defined as $S = \{(a,b): a, b \in R \text{ and } a \leq b^3\}$ is neither reflexive not symmetric not transitive.

31. If the function $f(x)$ given by $f(x) = \begin{cases} 3ax+b, & if\ x > 1 \\ 11, & if\ x = 1 \\ 5ax-2b, & if\ x < 1 \end{cases}$ is continuous at $x = 1$, then find the values of a and b.

OR

Find the value of k, so that the functions f defined by $f(x) = \begin{cases} kx+1, & if\ x \leq \pi \\ \cos x & if\ x > \pi \end{cases}$ is continuous at $x = \pi$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Find the intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36x}{5} + 11$ is

(i) strictly increasing
(ii) strictly decreasing.

OR

A ladder 5 m long is leaning against a wall. Bottom of ladder is pulled along the ground away from wall at the rate of 2 m/s. How fast is the height on the wall decreasing, when the foot of ladder is 4 m away from the wall?

33. Maximise and minimise $Z = x + 2y$ subject to the constraints
$x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x, y \geq 0$ Solve the above LPP graphically.

OR

A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours by a semi-skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer profit on an item of model A is Rs. 15 and on an items of model B is Rs. 10. How many of items of models should be made per day in order to maximize daily profit? Formulate the above LPP and solve it graphically and find the maximum profit.

34. (i) Evaluate $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$.

(ii) Write the value of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$.

(iii) Write the principal value of $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

OR

Using the principal value, evaluate the following:

(i) $\tan^{-1}1 + \sin^{-1}\left(-\frac{1}{2}\right)$

(ii) $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

35. Find the shortest distance between the following lines. $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$, $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$.

OR

Solve graphically the following system of in equations. $x + 2y \geq 20$, $3x + y \leq 15$

Section - E

Case study based questions are compulsory.

36. Different types of drugs affect your body in different ways, and the effects associated with drugs can vary from person to person. How a drug effects an individual is dependent on a variety of factors including body size, general health, the amount and strength of the drug, and whether any other drugs are in the system at the same time. It is important to remember that illegal drugs are not controlled substances, and therefore the quality and strength may differ from one batch to another.

The concentration $C(t)$ in milligrams per cubic centimeter (mg/cm^3) of a drug in a patient's bloodstream is $0.5\,\text{mg/cm}^3$ immediately after an injection and t minutes later is decreasing at the rate

$$C'(t) = \frac{-0.01e^{0.01t}}{(e^{0.01t}+1)^2}\ \text{mg/cm}^3 \text{ per minute}$$

A new injection is given when the concentration drops below $0.05\,\text{mg/cm}^3$.

(i) Find an expression for $C(t)$.

(ii) What is the concentration after 1 hour? After 3 hours?

37. The Vande Bharat Express, also known as Train 18, is a semi-high-speed, intercity, electric multiple unit train operated by the Indian Railways on 4 routes as of October 2022. Routes include New Delhi to Shri Mata Vaishno

In a survey at Vande Bharat Train, IRCTC asked the passenger to rate and review the food served in train. Suppose IRCTC asked 500 passenger selected at random to rate food according to price (low, medium, or high) and food (1, 2, 3, or 4 stars). The results of this survey are presented in the two-way, or contingency, table below. The numbers in this table represent frequencies. For example, in the third row and fourth column, 30 people rated the prices high and the food 4 stars. The last column contains the sum for each row, and similarly, the bottom row contains the sum for each column. These sums are often called marginal totals.

Price	**Food rating**			
	*	**	***	****
Low	20	30	90	10
Medium	50	80	90	30
High	20	10	40	30

Assume that these results are representative of the entire passenger of train, so the relative frequency of occurrence is the true probability of the event. A passenger from train is randomly selected.

(i) Find the probability that the passenger rates the prices medium.

(ii) Find the probability that the passenger rates the food 2 stars.

(iii) Suppose the passenger selected rates the prices high. What is the probability that he rates the restaurants 1 star?

(iv) Suppose the passenger selected does not rate the food 4 stars. What is the probability that she rates the prices high?

38. Publishing is the activity of making information, literature, music, software and other content available to the public for sale or for free. Traditionally, the term refers to the creation and distribution of printed works, such as books, newspapers, and magazines.

NODIA Press is a such publishing house having two branch at Jaipur. In each branch there are three offices. In each office, there are 2 peons, 5 clerks and 3 typists. In one office of a branch, 5 salesmen are also working. In each office of other branch 2 head-clerks are also working. Using matrix notations find:

(i) the total number of posts of each kind in all the offices taken together in each branch.

(ii) the total number of posts of each kind in all the offices taken together from both branches.

□□□□□□□

Sample Paper 15

Class - 12th Exam - 2024 - 25

Mathematics (Code-041)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided some questions.
9. Use of calculators is not allowed.

Section - A

Section A consists of 20 questions of 1 mark each.

1. If $A=\{1,2,3\}$ and $B=\{2,3,4\}$, then which of the following relations is a function from A to B?

(a) $\{(1,2),(2,3),(3,4),(2,2)\}$ (b) $\{(1,2),(2,3),(1,3)\}$

(c) $\{(1,3),(2,3),(3,3)\}$ (d) $\{(1,1),(2,3),(3,4)\}$

2. The order of the differential equation $\left[1+\left(\frac{dy}{dx}\right)\right]^{3/2}=\frac{d^2y}{dx^2}$ is

(a) 1 (b) 2

(c) 3 (d) 4

3. The function $f(x)=2+4x^2+6x^4+8x^6$ has

(a) only one maxima (b) only one minima

(c) no maxima and minima (d) many maxima and minima

4. $\int\frac{x^{e-1}+e^{x-1}}{x^e+e^x}dx$ is equal to

(a) $\log(x^e+e^x)+C$ (b) $e\log(x^e+e^x)+C$

(c) $\frac{1}{e}\log(x^e+e^x)+C$ (d) None of these

5. Area of the region satisfying $x\le 2$, $y\le|x|$ and $x\ge 0$ is

(a) 4 sq units (b) 1 sq unit

(c) 2 sq units (d) None of these

6. $2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$ holds good for all

(a) $|x| \leq 1$
(b) $1 \geq x \geq 0$
(c) $|x| \leq \frac{1}{\sqrt{2}}$
(d) none of these

7. The existence of the unique solution of the system of equations $x+y+z=\beta$; $5x-y+\alpha z=10$ and $2x+3y=6$ depends on

(a) α only
(b) β only
(c) Both α and β
(d) Neither β nor α

8. At $x=\frac{3}{2}$, the function $f(x)=\frac{|2x-3|}{2x-3}$ is

(a) continuous
(b) discontinuous
(c) differentiable
(d) non-zero

9. If $f(x)=\begin{cases}\frac{\log x}{x-1}, & if\ x \neq 1\\ k, & if\ x=1\end{cases}$ is continuous at $x=1$, then the value of k is

(a) 0
(b) -1
(c) 1
(d) e

10. Given two vectors $\vec{a}=2\hat{i}-3\hat{j}+6\hat{k}$, $\vec{b}=-2\hat{i}+2\hat{j}-\hat{k}$ and $\lambda=\frac{\text{the projection of } \vec{a} \text{ on } \vec{b}}{\text{the projectin of } \vec{b} \text{ on } \vec{a}}$ then the value of λ is

(a) $\frac{3}{7}$
(b) $\frac{7}{3}$
(c) 3
(d) 7

11. The integrating factor of the differential equation $\frac{dy}{dx}(x\log x)+y=2\log x$ is given by

(a) e^x
(b) $\log x$
(c) $\log\log x$
(d) x

12. If $dy/dx=e^{-2y}$ and $y=0$, when $x=5$, then the value of x, when $y=3$ is

(a) e^5
(b) e^6+1
(c) $\frac{e^6+9}{2}$
(d) $\log_e 6$

13. A constant force $\vec{F}=2\vec{i}-3\vec{j}+2\vec{k}$ is acting on a particle such that the particle is displaced from the point $A(1,2,3)$ to the point $B(3,4,5)$. The work done by the force is

(a) 2
(b) 3
(c) $\sqrt{17}$
(d) $2\sqrt{51}$

14. The points (5, 2, 4), $(6,-1,2)$ and $(8,-7,k)$ are collinear, if k is equal to

(a) -2
(b) 2
(c) 3
(d) -1

15. If x-coordinate of a point P of line joining the points and $R(5,2,-2)$ is 4, then the z-coordinate of P is

(a) -2 (b) -1

(c) 1 (d) 2

16. Minimum value of the function $f(x) = x^2 + x + 1$ is

(a) 1 (b) 3

(c) $\frac{3}{4}$ (d) 4

17. If $P(A) = \frac{1}{12}$, $P(B) = \frac{5}{12}$ and $P\left(\frac{B}{A}\right) = \frac{1}{15}$, then $P(A \cup B)$ is equal to

(a) $\frac{89}{180}$ (b) $\frac{90}{180}$

(c) $\frac{91}{180}$ (d) $\frac{92}{180}$

18. Range of the function $f(x) = \frac{x}{1+x^2}$ is

(a) $(-\infty, \infty)$ (b) $[-1, 1]$

(c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $[-\sqrt{2}, \sqrt{2}]$

19. **Assertion:** $I = \int_0^{\frac{\pi}{2}} \sqrt{\tan x}\, dx = \frac{\pi}{\sqrt{2}}$

Reason: $\tan x = t^2$ makes the integrand in I as a rational function.

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

20. **Assertion:** $f(\theta) = \begin{vmatrix} \cos(\theta+\alpha) & \cos(\theta+\beta) & \cos(\theta+\gamma) \\ \sin(\theta+\alpha) & \sin(\theta+\beta) & \sin(\theta+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix}$ is independent of θ

Reason: If $f(\theta) = c$ then $f(\theta)$ is independent of θ

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. What is the range of the function

$$f(x) = \frac{|x-1|}{x-1},\ x \neq 1?$$

22. If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 400$ and $|\vec{a}| = 5$, then write the value of $|\vec{b}|$.

OR

If $\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$, then find the projection of $\vec{a}$ on $\vec{b}$.

23. The probability distribution of a random variable X is given below

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
$P(X)$	p	$2p$	$3p$	$4p$	$5p$	$7p$	$8p$	$9p$	$10p$	$11p$	$12p$

Then, the value of p is

(a) $\frac{1}{72}$

(b) $\frac{3}{73}$

(c) $\frac{5}{72}$

(d) $\frac{1}{74}$

24. If a line makes angles 90°, 60° and θ with X, Y and Z-axis respectively, where θ is acute angle, then find θ.

25. Write the value of $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$.

OR

Write the value of $\int \frac{dx}{x^2 + 16}$.

Section - C

This section comprises of short answer-type questions (SA) of 3 marks each.

26. In the interval $\frac{\pi}{2} < x < \pi$, find the value of x for which the matrix $\begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$ is singular.

27. Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \frac{5}{13}$ and $P\left(\frac{A}{B}\right) = \frac{2}{5}$.

28. If $e^y (x + 1) = 1$, then show that $\frac{d^2 y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

29. Write the value of $\cos^{-1}\left(\cos \frac{7\pi}{6}\right)$.

30. Evaluate $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx$.

OR

Evaluate $\int_0^\pi \frac{x \tan x}{\sec x + \tan x} dx$.

31. The sides of an equilateral triangle are increasing at the rate of 2 cm/s. Find the rate at which the area increases, when the side is 10 cm?

OR

Find the value(s) of x for which $y = [x(x - 2)]^2$ is an increasing function.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Evaluate $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx$.

33. Maximise and minimise $Z = x + 2y$ subject to the constraints
$x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x, y \geq 0$ Solve the above LPP graphically.

OR

Maximise $Z = 8x + 9y$ subject to the constraints given below $2x + 3y \leq 6$, $3x - 2y \leq 6$, $y \leq 1$, $x, y \geq 0$.

34. Solve the following differential equation. $\left[y - x\cos\left(\frac{y}{x}\right)\right]dy + \left[y\cos\left(\frac{y}{x}\right)2x\sin\left(\frac{y}{x}\right)\right]dx = 0$

OR

Find the particular solution of the differential equation $(3xy + y^2)\,dx + (x^2 + xy)\,dy = 0$, for $x = 1$ and $y = 1$.

35. Find the shortest distance between the lines
$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$.

OR

Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

Section - E

Case study based questions are compulsory.

36. Sachin Mehara is a final year student of civil engineering at IIT Delhi. As a final year real time project, he has got the job of designing a auditorium for cultural activities purpose. The shape of the floor of the auditorium is rectangular and it has a fixed perimeter, say. P.

Based on the above information, answer the following questions.

(i) If l and b represents the length and breadth of the rectangular region, then find the relationship between l ,b, p.

(ii) Find the area (A) of the floor, as a function of is l

(iii) College authority is interested in maximising the area of the floor A. For this purpose, find the value of l.

OR

Find the maximum area of the floor.

37. Pfizer Inc. is an American multinational pharmaceutical and biotechnology corporation headquartered on 42nd Street in Manhattan, New York City. The company was established in 1849 in New York by two German immigrants, Charles Pfizer and his cousin Charles F. Erhart. Pfizer develops and produces medicines and vaccines for immunology, oncology, cardiology, endocrinology, and neurology.

The purchase officer of the Pfizer informs the production manger that during the month, following supply of three chemicals, Asprin (A), Caffieine (C) and Decongestant (D) used in the production of three types of pain-killing tablet will be 16, 10 and 16 kg respectively. According to the specification, each strip of 10 tables of Paingo requires 2 gm of A, 3 gm of C and 1 gm of D. The requirements for other tables are:

X-prene	4 gm of A	1 gm of C	3 gm of D
Relaxo	1 gm of A	2 gm of C	3 gm of D

(i) Taking the suitable variable form the system of equation that represent given problem.
(ii) Use matrix inversion method to find the number of strips of each type so that raw materials are consumed entirely.

38. At its simplest, a fair die states that each of the faces has a similar probability of landing facing up. A standard fair six-sided die, for example, can be regarded as "fair" if each of the faces consists of a probability of 1/6.

A fair die is rolled. Consider the events $A = \{1,3,5\}$, $B = \{2,3\}$, and $C = \{2,3,4,5,\}$
On the basis of above information, answer the following questions.
(i) Find the probability $P(A/B)$ and $P(B/A)$.
(ii) Find the probability $P(A/C)$, $P(A \cap B/C)$ and $P(A \cup B/C)$

Sample Paper 16

Class - 12th Exam - 2024 - 25

Mathematics (Code-041)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided some questions.
9. Use of calculators is not allowed.

SECTION - A

Section A consists of 20 questions of 1 mark each.

1. If $\int \frac{x+2}{2x^2+6x+5}dx = P\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{2}\int \frac{dx}{2x^2+6x+5}$, then the value of P is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) 2

2. $\int_0^{\pi/2} \left|\cos\left(\frac{x}{2}\right)\right| dx$ is equal to

(a) 1 (b) -2

(c) $\sqrt{2}$ (d) 0

3. An integrating factor of the differential equation $x\frac{dy}{dx} + y\log x = xe^x x^{-\frac{1}{2}\log x} (x > 0)$ is

(a) $x^{\log x}$ (b) $(\sqrt{x})^{\log x}$

(c) $(\sqrt{e})^{(\log x)^2}$ (d) e^{x^2}

4. For the value of λ is the function defined by $f(x) = \begin{cases} \lambda(x^2-2x), & \text{if } x \le 0 \\ 4x+1, & \text{if } x < 0 \end{cases}$ continuous at $x = 0$?

(a) 0 (b) 1

(c) 2 (d) Not continuous at $x = 0$ for any value of λ

5. If gas is being pumped into a spherical balloon at the rate of 30 ft3/min. Then, the rate at which the radius increases, when it reaches the value 15 ft is

(a) $\frac{1}{15\pi}$ ft/min (b) $\frac{1}{30\pi}$ ft/min

(c) $\frac{1}{20}$ ft/min (d) $\frac{1}{25}$ ft/min

6. The length of the longest interval, in which $f(x) = 3\sin x - 4\sin^3 x$ is increasing is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{2}$ (d) π

7. If $A(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then the matrix $A^2(\alpha)$ is equal to

(a) $A(2\alpha)$ (b) $A(\alpha)$

(c) $A(3\alpha)$ (d) $A(4\alpha)$

8. If $f(x) = \begin{cases} ax + 3, & x \le 2 \\ a^2 x - 1, & x > 2 \end{cases}$, then the values of a for which f is continuous for all x are

(a) 1 and -2 (b) 1 and 2

(c) -1 and 2 (d) -1 and -2

9. Area of the region satisfying $x \le 2$, $y \le |x|$ and $x \ge 0$ is

(a) 4 sq units (b) 1 sq unit

(c) 2 sq units (d) None of these

10. The area bounded by the parabola $y^2 = 8x$ and its latusrectum is

(a) 16/3 sq units (b) 32/3 sq units

(c) 8/3 sq units (d) 64/3 sq units

11. Solution of $e^{dy/dx} = x$, when $x = 1$ and $y = 0$ is

(a) $y = x(\log x - 1) + 4$ (b) $y = x(\log x - 1) + 3$

(b) $y = x(\log x + 1) + 1$ (d) $y = x(\log x - 1) + 1$

12. The figure formed by four points $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 3\hat{j}$, $3\hat{i} - 5\hat{j} - 2\hat{k}$, $\hat{k} - \hat{j}$ is a

(a) parallelogram (b) rectangle

(c) trapezium (d) square

13. If $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} + \vec{b}$ makes an angle of $60°$ with $\vec{a}$, then

(a) $|\vec{a}| = 2|\vec{b}|$ (b) $2|\vec{a}| = |\vec{b}|$

(c) $|\vec{a}| = \sqrt{3}|\vec{b}|$ (d) $\sqrt{3}|\vec{a}| = |\vec{b}|$

14. If the direction cosines of a line are $\left(\frac{1}{c}, \frac{1}{c}, \frac{1}{c}\right)$, then

(a) $0 < c < 1$

(b) $c > 2$

(c) $c = \pm\sqrt{2}$

(d) $c = \pm\sqrt{3}$

15. A coin and six faced die, both unbiased, are thrown simultaneously. The probability of getting a head on the coin and an odd number on the die is

(a) $\frac{1}{2}$

(b) $\frac{3}{4}$

(c) $\frac{1}{4}$

(d) $\frac{2}{3}$

16. Find the angle between the following pairs of lines:

$$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$$

and

$$\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$$

(a) $\cos^{-1}\left(\frac{19}{21}\right)$

(b) $\cos^{-1}\left(\frac{16}{21}\right)$

(c) $\cos^{-1}\left(\frac{13}{21}\right)$

(d) $\cos^{-1}\left(\frac{11}{21}\right)$

17. If $P(A) = \frac{1}{12}$, $P(B) = \frac{5}{12}$ and $P\left(\frac{B}{A}\right) = \frac{1}{15}$, then $P(A \cup B)$ is equal to

(a) $\frac{89}{180}$

(b) $\frac{90}{180}$

(c) $\frac{91}{180}$

(d) $\frac{92}{180}$

18. Solution of the differential equation $xdy - ydx = 0$ represents a

(a) parabola

(b) circle

(c) hyperbola

(d) straight line

19. If $n > 1$, then

Assertion : $\int_0^\infty \frac{dx}{1+x^n} = \int_0^1 \frac{dx}{(1-x^n)^{1/n}}$

Reason : $\int_a^b f(x)\,dx = \int_a^b f(a+b+x)\,dx$

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

Continue on next page......

20. **Assertion :** $\int_{-\pi/2}^{\pi/2} |\sin x|\, dx = 2$

Reason : $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$, where c $\varepsilon a, b)$

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

SECTION - B

Section B consists of 5 questions of 2 marks each.

21. Find the general solution of equation $\frac{dy}{dx} + y = 1 (y \neq 1)$

22. If $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, then find a unit vector parallel to the vector $\vec{a} + \vec{b}$.

23. Determine the value of constant k so that the function $f(x) = \begin{cases} \frac{kx}{|x|}, & if\ x < 0 \\ 3, & if\ x \geq 0 \end{cases}$ is continuous at $x = 0$.

24. Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 + \sin\theta & 1 \\ 1 & 1 & 1 + \cos\theta \end{vmatrix}$

25. Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \frac{5}{13}$ and $P\left(\frac{A}{B}\right) = \frac{2}{5}$.

Continue on next page......

SECTION - C

Section C consists of 6 questions of 3 marks each.

26. Find the general solution of equation $y\log y\,dx - x\,dy = 0$

27. Find $\int \frac{x-3}{(x-1)^3} e^x\,dx$.

28. Check whether the relation R in the set R of real numbers, defined by $R = \{(a,b): 1 + ab > 0\}$, is reflexive, symmetric or transitive.

29. If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the identity matrix of order 2, then show that $A^2 = 4A - 3I$. Hence, find A^{-1}.

30. Find the position of a point R, which divides the line joining two points P and Q whose position vectors are $2\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ respectively, externally in the ratio 1 : 2. Also, show that P is the mid-point of line segment RQ.

31. Find the value of k, for which $f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & if -1 \leq x < 0 \\ \frac{2x+1}{x-1}, & if\ 0 \leq x < 1 \end{cases}$ is continuous at $x = 0$.

SECTION - D

Section D consists of 4 questions of 5 marks each.

32. AB is the diameter of a circle and C is any point of the circle. Show that the area of ΔABC is maximum, when it is an isosceles triangle.

33. Solve graphically.
$x + y \geq 5$, $2x + 3 \geq 3y$, $0 \leq x \leq 4$, $0 \leq y \leq 2$.

Continue on next page......

CLICK HERE TO SEE ANSWERS

34. Write the following functions in the simplest from: $\tan^{-1}\frac{\sqrt{1+x^2}}{x}$, $x \neq 0$

OR

Find the value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.Thinking Process

Use the property, $\tan^{-1}\tan x = x$, $x \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and $\cos^{-1}(\cos x) = x$, $x \in [0,\pi]$ to get the answer.

35. Find the shortest distance between the lines

$$\vec{r} = \left(\hat{i}+2\hat{j}+\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$$

and $$\vec{r} = \left(2\hat{i}-\hat{j}-\hat{k}\right)+\mu\left(2\hat{i}+\hat{j}+2\hat{k}\right)$$

SECTION - E

Case study based questions are compulsory.

36. Bata India is the largest retailer and leading manufacturer of footwear in India and is a part of the Bata Shoe Organization. Incorporated as Bata Shoe Company Private Limited in 1931, the company was set up initially as a small operation in Konnagar (near Calcutta) in 1932. In January 1934,

The manager of BATA show room at Jaipur determines that the price p (dollars) for each pair of a popular brand of sports sneakers is changing at the rate of

$$p'(x) = \frac{-300x}{(x^2+9)^{3/2}}$$

when x (hundred) pairs are demanded by consumers. When the price is \$75 per pair, 400 pairs ($x = 4$) are demanded by consumers.

(i) Find the demand (price) function $p(x)$.

(ii) At what price will 500 pairs of sneakers be demanded? At what price will no sneakers be demanded?

(iii) How many pairs will be demanded at a price of \$90 per pair?

37. Goods and Services Tax (GST) is an indirect tax (or consumption tax) used in India on the supply of goods and services. It is a comprehensive, multistage, destination-based tax: comprehensive because it has subsumed almost all the indirect taxes except a few state taxes. Multi-staged as it is, the GST is imposed at every step in the production process, but is meant to be refunded to all parties in the various stages of production other than the final consumer and as a destination-based tax, it is collected from point of consumption and not point of origin like previous taxes.

A GST form is either filed on time or late, is either from a small or a large business, and is either accurate or inaccurate. There is an 11% probability that a form is from a small business and is accurate and on time. There is a 13% probability that a form is from a small business and is accurate but is late. There is a 15% probability that a form is from a small business and is on time. There is a 21% probability that a form is from a small business and is inaccurate and is late.

(i) If a form is from a small business and is accurate, what is the probability that it was filed on time?

(ii) What is the probability that a form is from a large business?

38. Fertilizer, natural or artificial substance containing the chemical elements that improve growth and productiveness of plants. Fertilizers enhance the natural fertility of the soil or replace chemical elements taken from the soil by previous crops.

The following matrix gives the proportionate mix of constituents used for three fertilisers:

$$\begin{array}{r} \\ \text{I} \\ \text{Fertilisers II} \\ \text{III} \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \quad \text{Constituents} \\ \begin{bmatrix} 0.5 & 0 & 0.5 & 0 \\ 0.2 & 0.3 & 0 & 0.5 \\ 0.2 & 0.2 & 0.1 & 0.5 \end{bmatrix} \end{array}$$

(i) If sales are 1000 tins (of one kilogram) per week, 20% being fertiliser I, 30% being fertiliser II and 50% being fertiliser III, how much of each constituent is used.

(ii) If the cost of each constituents is 5, 6, 7.5 and 10 per 100 grams, respectively, how much does a one kilogram tin of each fertiliser cost

(iii) What is the total cost per week?

□□□□□□□

Sample Paper 17

Class - 12th Exam - 2024 - 25

Mathematics (Code-041)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided some questions.
9. Use of calculators is not allowed.

SECTION - A

Section A consists of 20 questions of 1 mark each.

1. If A is 3×4 matrix and B is a matrix such that $A'B$ and BA' are both defined, then B is of the type

(a) 4×4 (b) 3×4

(c) 4×3 (d) 3×3

2. $\int \frac{\sec^2(\sin^{-1}x)}{\sqrt{1-x^2}}dx$ is equal to

(a) $\sin(\tan^{-1}x)+C$ (b) $\tan(\sec^{-1}x)+C$

(c) $\tan(\sin^{-1}x)+C$ (d) $-\tan(\cos^{-1}x)+C$

3. For all real values of x, the minimum value of $\frac{1-x+x^2}{1+x+x^2}$ is

(a) 0 (b) 1

(c) 3 (d) $\frac{1}{3}$

4. $\int\sqrt{1+\cos x}\,dx$ is equal to

(a) $2\sin\left(\frac{x}{2}\right)+C$ (b) $\sqrt{2}\sin\left(\frac{x}{2}\right)+C$

(c) $2\sqrt{2}\sin\left(\frac{x}{2}\right)+C$ (d) $\frac{1}{2}\sin\left(\frac{x}{2}\right)+C$

5. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then

(a) $A^2 + 7A - 5I = O$

(b) $A^2 - 7A + 5I = O$

(c) $A^2 + 5A - 7I = O$

(d) $A^2 - 5A + 7I = O$

6. The probability distribution of a random variable X is given below

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
$P(X)$	p	$2p$	$3p$	$4p$	$5p$	$7p$	$8p$	$9p$	$10p$	$11p$	$12p$

Then, the value of p is

(a) $\frac{1}{72}$

(b) $\frac{3}{73}$

(c) $\frac{5}{72}$

(d) $\frac{1}{74}$

7. If $P = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix}$ and $Q = PP^T$, then the value of Q is

(a) 2

(b) -2

(c) 1

(d) 0

8. The derivative of $\cos^3 x$ with respect to $\sin^3 x$ is

(a) $-\cot x$

(b) $\cot x$

(c) $\tan x$

(d) $-\tan x$

9. The maximum value of xe^{-x} is

(a) e

(b) $1/e$

(c) $-e$

(d) $-1/e$

10. If $\sin(x+y) = \log(x+y)$, then dy/dx is equal to

(a) 2

(b) -2

(c) 1

(d) -1

11. The area bounded by the parabola $y^2 = 8x$ and its latusrectum is

(a) $16/3$ sq units

(b) $32/3$ sq units

(c) $8/3$ sq units

(d) $64/3$ sq units

12. Evaluate $\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$

(a) $\frac{x^3}{3} + x + C$

(b) $\frac{x^3}{4} + x + C$

(c) $\frac{x^3}{5} + x + C$

(d) $\frac{x^3}{6} + x + C$

13. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle $\pi/3$, then the value of $|\vec{a} + \vec{b}|$ is

(a) equal to

(b) greater than 1

(c) equal to

(d) less than 1

14. The point of intersection of lines $\frac{x-5}{3}=\frac{y-7}{-1}=\frac{z+2}{1}$ and $\frac{x+3}{-36}=\frac{y-3}{2}=\frac{z-6}{4}$ is

(a) $(5,7,-2)$ (b) $(-3,3,6)$

(c) $(2,10,4)$ (d) $\left(21,\frac{5}{3},\frac{10}{3}\right)$

15. The area bounded by $y=\log x$, X-axis and ordinates $x=1$, $x=2$ is

(a) $\frac{1}{2}(\log 2)^2$ (b) $\log(2/e)$

(c) $\log(4/e)$ (d) $\log 4$

16. It is given that the events A and B are such that $P(A)=\frac{1}{4}$, $P(A/B)=\frac{1}{2}$ and $P(B/A)=\frac{2}{3}$. Then, $P(B)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

17. If $\vec{a}\cdot\vec{b}=-|\vec{a}||\vec{b}|$, then the angle between a and b is

(a) 45° (b) 180°

(c) 90° (d) 60°

18. Equation of the line passing through $(2,-1,1)$ and parallel to the line $\frac{x-5}{4}=\frac{y+1}{-3}=\frac{z}{5}$ is

(a) $\frac{x-2}{4}=\frac{y+1}{-3}=\frac{z-1}{5}$ (b) $\frac{x-2}{4}=\frac{y+1}{3}=\frac{z-1}{5}$

(c) $\frac{x-2}{-4}=\frac{y+1}{-3}=\frac{z-1}{5}$ (d) None of the above

19. **Assertion :** The general solution of $\frac{dy}{dx}+y=1$ is $ye^x=e^x+c$

Reason : The number of arbitrary constants is in the general solution of the differential equation is equal to the order of differential equation.

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

20. **Assertion :** The elimination of four arbitrary constants in $y=(c_1+c_2+c_3e^{c_4})x$ results into a differential equation of the first order $x\frac{dy}{dx}=y$

Reason : Elimination of n arbitrary constants requires in general a differential equation of the n^{th} order.

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

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SECTION - B

Section B consists of 5 questions of 2 marks each.

21. If a line has direction ratios $(2, -1, -2)$, then what are its direction cosines?

22. Prove that if E and F are independent events, then the events E and F' are also independent.

23. The total cost $C(x)$ associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$.

Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

24. Find the general solution of the differential equation $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

25. Minimize $Z = 3x + 5y$ such that $x + 3y \geq 3$, $x + y \geq 2$, x, $y \geq 0$.

SECTION - C

Section C consists of 6 questions of 3 marks each.

26. Find a matrix A such that $2A - 3B + 5C = O$, where $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$.

27. Solve graphically the following system of in equations.
$x + 2y \geq 20$, $3x + y \leq 15$

28. Find $\int \frac{2\cos x}{(1-\sin x)(2-\cos^2 x)}dx$.

29. Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$.

30. Find the vector and cartesian equations of the line through the point $(1, 2, -4)$ and perpendicular to the lines $\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

31. A ladder 5 m is leaning against a wall .The bottom of the ladder is pulled along the ground anyway from the wall, at the rate 2 cm/s. How fast is its height on the wall decreasing, when the foot of the ladder is 4 meter away from the wall?

SECTION - D

Section D consists of 4 questions of 5 marks each.

32. If $y = x^x$, then prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$.

33. Let $A = \{x \in Z : 0 \le x \le 12\}$. Show that $R = \{(a, b) : a, b \in A, |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2].

34. The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ and hence find unit vector along $\vec{b} \times \vec{c}$.

35. Using integration, find the area of the following region.
$\{(x, y) : |x - 1| \le y \le \sqrt{5 - x^2}\}$

SECTION - E

Case study based questions are compulsory.

36. Joint Entrance Examination – Advanced, is an academic examination held annually in India. It is organised by one of the seven zonal IITs under the guidance of the Joint Admission Board on a round-robin rotation pattern for the qualifying candidates of the JEE-Main. A candidate can attempt JEE (Advanced) maximum of two times in two consecutive years. A successful candidates get the admission in any IITs of India based on merit.

Applicants have a 0.26 probability of passing IIT advanced test when they take it for the first time, and if they pass it they can get admission in IIT. However, if they fail the test the first time, they must take the test a second time, and when applicants take the test for the second time there is a 0.43 chance that they will pass and be allowed to get admission. Applicants are rejected if the test is failed on the second attempt.

(i) What is the probability that an applicant gets admission in IIT but needs two attempts at the test?

(ii) What is the probability that an applicant gets admission in IIT?

(iii) If an applicant gets admission in IIT, what is the probability that he or she passed the test on the first attempt?

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37. When it comes to taxes, there are two types of taxes in India - Direct and Indirect tax. The direct tax includes income tax, gift tax, capital gain tax, etc while indirect tax includes goods and service tax i.e. GST and any local tax.

A company earns before tax profits of ₹100000. It is committed to making a donation to the Red Cross 10% of its after-tax profits. The Central Government levies income taxes of 50% of profits after deducing charitable donations and any local taxes. The company must also pay local taxes of 10% of its profit less the donation to the Red Cross.

(i) Taking the suitable variable form the system of equation that represent given problem.

(ii) Compute how much the company pays in income taxes, local taxes and as a donation to the Red Cross, using Cramer's Rule.

38. Chemical reaction, a process in which one or more substances, the reactants, are converted to one or more different substances, the products. Substances are either chemical elements or compounds. A chemical reaction rearranges the constituent atoms of the reactants to create different substances as products.

In a certain chemical reaction, a substance is converted into another substance at a rate proportional to the square of the amount of the first substance present at any time t. Initially $(t = 0)$ 50 g of the first substance was present; 1 hr later, only 10 g of it remained.

(i) Find an expression that gives the amount of the first substance present at any time t.

(ii) What is the amount present after 2 hr?

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Sample Paper 18

Class - 12th Exam - 2024 - 25

Mathematics (Code-041)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided some questions.
9. Use of calculators is not allowed.

SECTION - A

Section A consists of 20 questions of 1 mark each.

1. The length of the longest interval, in which $f(x) = 3\sin x - 4\sin^3 x$ is increasing is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{2}$ (d) π

2. $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ is equal to

(a) 0 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

3. If $f(\theta) = \begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$, then for all Values of θ

(a) $f(\theta) = 0$ (b) $f(\theta) = 1$

(c) $f(\theta) = -1$ (d) None of these

4. If $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and A^2 is the identity matrix, then x is Equal to

(a) -1 (b) 0

(c) 1 (d) 2

5. If $f:[0,\frac{\pi}{2}]\to[0,\infty]$ be a function defined by $y=\sin(\frac{\pi}{2})$, Then f is

(a) injective (b) subjective

(c) bijective (d) none of these

6. $3a\int_{0}^{1}\left(\frac{ax-1}{a-1}\right)^2 dx$ is equal to

(a) $a-1+(a-1)^{-2}$ (b) $a+a^{-2}$

(c) $a-a^2$ (d) $a^2+\frac{1}{a^2}$

7. The area of a parallelogram whose adjacent sides are $\hat{i}-2\hat{j}+3\hat{k}$ and $2\hat{i}+\hat{j}-4\hat{k}$, is

(a) $5\sqrt{3}$ (b) $10\sqrt{3}$

(c) $5\sqrt{6}$ (d) $10\sqrt{6}$

8. The order of the differential equation of all conics whose centre lie at the origin is given by

(a) 2 (b) 3

(c) 4 (d) 5

9. A mapping $f:n\to N$, where N is the set of natural numbers is define as

$$f(n)=\begin{cases} n^2, & \text{for } n \text{ odd} \\ 2n+1, & \text{for } n \text{ even}\end{cases}$$ For $n\in N$. Then, f is

(a) Subjective but not injective

(b) Injective but not subjective

(c) Bijective

(d) neither injective nor subjective

10. Solution of $(x+2y^3)\,dy=y\,dx$ is

(a) $x=y^3+cy$ (b) $x+y^3=cy$

(c) $y^2-x=cy$ (d) none of these

11. If $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{p}$ is a non-zero vector such that $p\vec{r}+(\vec{r}\cdot\vec{b})\vec{a}=\vec{c}$, Then $\vec{r}$ is equal to

(a) $\frac{\vec{c}}{p}-\frac{(\vec{b}\cdot\vec{c})\vec{a}}{p^2}$ (b) $\frac{\vec{a}}{p}-\frac{(\vec{c}\cdot\vec{a})\vec{b}}{p^2}$

(c) $\frac{\vec{b}}{p}-\frac{(\vec{a}\cdot\vec{b})\vec{c}}{p^2}$ (d) $\frac{\vec{c}}{p^2}-\frac{(\vec{b}\cdot\vec{c})\vec{a}}{p}$

12. The lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x}{2}=\frac{y+2}{2}=\frac{z-3}{-2}$ are

(a) parallel (b) intersecting

(c) at right angle (d) none of these

Continue on next page......

13. A coin and six faced die, both unbiased, are thrown simultaneously. The probability of getting a head on the coin and an odd number on the die is

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$

(c) $\frac{1}{4}$ (d) $\frac{2}{3}$

14. The number of solutions of $y' = \frac{y+1}{x-1}, y(1) = 2$ is

(a) zero (b) one

(c) two (d) infinite

15. Solution set of the inequality $x \geq 0$ is

(a) half plane on the left of y-axis

(b) half plane on the right of y-axis excluding the points of y-axis

(c) half plane on the right of y-axis including the points on y-axis

(d) none of the above

16. Find the area of a curve $xy = 4$, bounded by the lines $x = 1$ and X-axis.

(a) $\log 12$ (b) $\log 64$

(c) $\log 81$ (d) $\log 27$

17. The derivative of $\cos^3 x$ with respect to $\sin^3 x$ is

(a) $-\cot x$ (b) $\cot x$

(c) $\tan x$ (d) $-\tan x$

18. Let $f(x) = x - \cos x, x \varepsilon R$, then f is

(a) a decreasing function (b) an odd function

(c) an increasing function (d) none of the above

19. **Assertion:** $\int \sin 3x \cos 5x \, dx = \frac{-\cos 8x}{16} + \frac{\cos 2x}{4} + C$

Reason: $2\cos A \sin B = \sin(A+B) - \sin(A-B)$

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

20. **Assertion:** If $x = at^2$ and $y = 2at$, then $\left.\frac{d^2y}{dx^2}\right|_{t=2} = \frac{-1}{16a}$

Reason: $\frac{d^2y}{dx^2} = \left(\frac{dy}{dt}\right)^2 \times \left(\frac{dt}{dx}\right)^2$

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

CLICK HERE TO SEE ANSWERS

SECTION - B

Section B consists of 5 questions of 2 marks each.

21. Find $|\vec{x}|$, if for a unit vector $\hat{a}$, $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a}) = 15$.

OR

If $\vec{a} = 2\hat{i}+3\hat{j}+\hat{k}$, $\vec{b} = \hat{i}-2\hat{j}+\hat{k}$ and $\vec{c} = -3\hat{i}+\hat{j}+2\hat{k}$, find $[\vec{a}\,\vec{b}\,\vec{c}]$.

22. If $A = \{1,2,3\}, B = \{4,5,6,7\}$ and $f = \{(1,4),(2,5),(3,6)\}$ is a function from A to B. State whether f is one & one or not.

23. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(\frac{A}{B}) = \frac{1}{2}$ and $P(\frac{B}{A}) = \frac{2}{3}$. Then, find $P(B)$?

OR

Find the probability distribution of X, the number of heads in a simultaneous toss of two coins.

24. Find the vector equation of the line passing through the point $A(1,2,-1)$ and parallel to the line $5x-25 = 14-7y = 35z$.

25. Evaluate $\int (ax-b)^3 dx$.

OR

Evaluate $\int \frac{(1+\log x)^2}{x} dx$.

SECTION - C

Section C consists of 6 questions of 3 marks each.

26. Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$

27. Show that the function $f(x) = x^3-3x^2+3x$, $x \in R$ is increasing on R.

OR

Find the intervals in which the function $f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$ is
(i) strictly increasing
(ii) strictly decreasing.

28. Let $\vec{a} = 4\hat{i}+5\hat{j}-\hat{k}$, $\vec{b} = \hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}-\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d}\cdot\vec{a} = 21$.

Continue on next page......

29. Evaluate $\int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}\,dx$.

OR

Evaluate $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x}\,dx$.

30. Write the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$.

31. If $(x-y)\,e^{\frac{x}{x-y}} = a$, prove that $y\frac{dy}{dx} + x = 2y$.

SECTION - D

Section D consists of 4 questions of 5 marks each.

32. Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$, given that $y = 0$, when $x = 0$.

OR

Find the particular solution of the differential equation $x(1+y^2)\,dx - y(1+x^2)\,dy = 0$, given that $y = 1$, when $x = 0$.

33. Maximize $Z = 3x + 2y$ subject to $x + 2y \leq 0$, $3x + y \leq 15$, x, $y \geq 0$.

OR

Minimise $Z = x + 2y$ subject to $2x + y \geq 3$, $x + 2y \geq 6$, x, $y \geq 0$. Show that the minimum of Z occurs at more than two points.

34. If $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2$, $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5$ and $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$

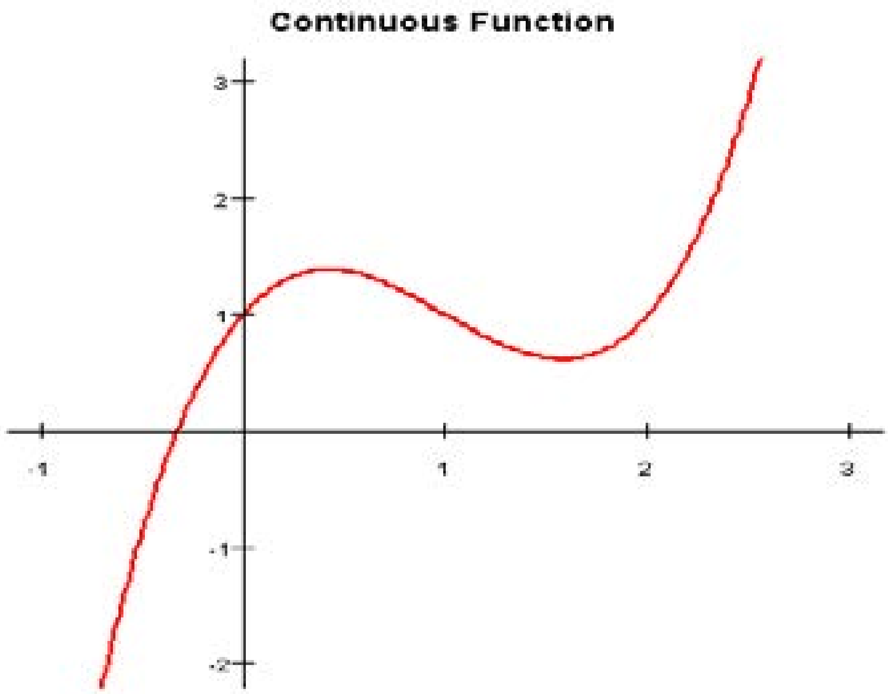
Continue on next page......

35. Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Also, find their point intersection.

SECTION - E

Case study based questions are compulsory.

36. In mathematics, a continuous function is a function such that a continuous variation of the argument induces a continuous variation of the value of the function. This means that there are no abrupt changes in value, known as discontinuities.



Let $f(x)$ be a continuous function defined on $[a,b]$, then $\int_a^b f(x)\,dx=\int_0^b f(a+b-x)\,dx$ On the basis of above information, answer the following questions.

(i) Evaluate $\int_1^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}}\,dx$

(ii) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \log \tan x\,dx$

(iii) Evaluate $\int_a^b \frac{x^{\frac{1}{n}}}{x^{\frac{1}{n}}+(a+b-x)^{\frac{1}{n}}}\,dx]$

OR

Evaluate $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}\,dx]$

Continue on next page......

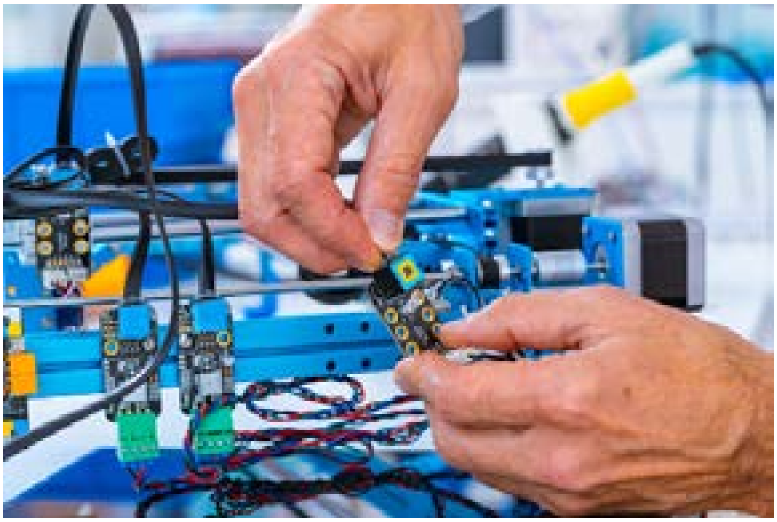
37. Bob is taking a learning test in which the time he takes to memorize items from a given list is recorded. Let $M(t)$ be the number of items he can memorize in t minutes. His learning rate is found to be

$$M'(t) = 0.4t - 0.005t^2$$

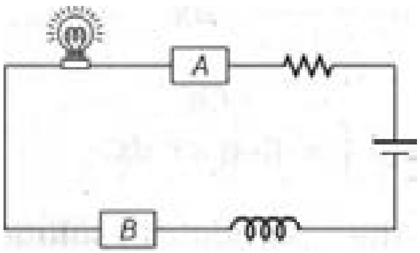
(i) How many items can Bob memorize during the first 10 minutes?

(ii) How many additional items can he memorize during the next 10 minutes (from time $t = 10$ to $t = 20$)?

38. An electro-mechanical assembler basically makes machines or/and other assemblies that contain electronic components like wires or microchips. Typically assemblers use blueprints, work instructions, and computer software to manufacture whatever they are working on.



An electronic assembly consists of two sub-systems say A and B as shown below.



From previous testing procedures, the following probabilities are assumed to be known $P(A\,\text{fails}) = 0.2$, $P(B\,\text{fails alone}) = 0.15$, $P(A\,\text{and}\,B\,\textit{fail}) = 0.15$.

On the basis of above information, answer the following questions.

(i) Find the probability P(B fails) and the probability P(A fails alone).

(ii) Find the probability P(whole system fail) and the probability P(A fails/B has failed).

□□□□□□□

Sample Paper 19

Class - 12th Exam - 2024 - 25

Mathematics (Code-041)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided some questions.
9. Use of calculators is not allowed.

SECTION - A

Section A consists of 20 questions of 1 mark each.

1. If $y = a\log|x| + bx^2 + x$ has its extreme value at $x = -1$ and $x = 2$, then

(a) $a = 2, b = -1$ (b) $a = 2, b = -\frac{1}{2}$

(c) $a = -2, b = \frac{1}{2}$ (d) $a = -2, b = -\frac{1}{2}$

2. The point of inflexion for the curve $y = x^{5/3}$ is

(a) $(1,1)$ (b) $(0,0)$

(c) $(1,0)$ (d) $(0,1)$

3. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined to x-axis at angles $30°$ and $120°$, then $|\vec{a} \times \vec{b}|$ equals

(a) $\sqrt{\frac{2}{3}}$ (b) $\sqrt{2}$

(c) $\sqrt{3}$ (d) 2

4. $\int_0^{\pi/2} \left|\cos\left(\frac{x}{2}\right)\right| dx$ is equal to

(a) 1 (b) -2

(c) $\sqrt{2}$ (d) 0

5. The point of discontinuous of $\tan x$ are

(a) $n\pi,\ n \in I$ (b) $2n\pi,\ n \in I$

(c) $(2n+1)\frac{\pi}{2},\ n \in I$ (d) None of these

6. The area bounded by the curve $y = \frac{1}{2}x^2$, the X-axis and the ordinate $x = 2$ is

(a) $\frac{1}{3}$ sq unit
(b) $\frac{2}{3}$ sq unit
(c) 1 sq unit
(d) $\frac{4}{3}$ sq unit

7. If A and B are two equivalence relations defined on set C, then

(a) $A \cap B$ is an equivalence relation
(b) $A \cap B$ is not an equivalence relation
(c) $A \cap B$ is an equivalence relation
(d) $A \cap B$ is not an equivalence relation

8. The range of the function $f(x) = x^2 + 2x + 2$ is

(a) $(1, \infty)$
(b) $(2, \infty)$
(c) $(0, \infty)$
(d) $[1, \infty)$

9. If $x, y, z \varepsilon R$ and $x + y + z = xyz$, then the value of $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$ is

(a) π
(b) $\frac{\pi}{2}$
(c) $\frac{3\pi}{2}$
(d) $\frac{\pi}{4}$

10. The value of k such that the lines $2x - 3y + k = 0$, $3x - 4y - 13 = 0$ and $8x - 11y - 33 = 0$ are concurrent, is

(a) 20
(b) -7
(c) 7
(d) -20

11. If $\int \frac{x+2}{2x^2+6x+5}dx = P\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{2}\int \frac{dx}{2x^2+6x+5}$, then the value of P is

(a) $\frac{1}{3}$
(b) $\frac{1}{2}$
(c) $\frac{1}{4}$
(d) 2

12. The solution of the equation $(x^2 - xy)\,dy = (xy + y^2)\,dx$ is

(a) $xy = ce^{-y/x}$
(b) $xy = ce^{-x/y}$
(c) $yx^2 = ce^{1/x}$
(d) none of these

13. The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents

(a) straight line
(b) circle
(c) parabola
(d) ellipse

14. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is

(a) $x + y = \frac{x^2}{2} + c$
(b) $x - y = \frac{1}{3}x^3 + c$
(c) $xy = \frac{1}{4}x^4 + c$
(d) $y - x = \frac{1}{4}x^4 + c$

15. Let D, E, F are the mid points of sides BC, CA, AB respectively of $\triangle ABC$. Which of the following is true?

(a) $\overrightarrow{AB} = 2\overrightarrow{ED}$
(b) $\overrightarrow{AB} = 2\overrightarrow{DE}$
(c) $\overrightarrow{AB} = \overrightarrow{ED}$
(d) $\overrightarrow{AB} = 2\overrightarrow{DF}$

16. The points (5, 2, 4), $(6, -1, 2)$ and $(8, -7, k)$ are collinear, if k is equal to

(a) -2
(b) 2
(c) 3
(d) -1

17. If A and B are two independent events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$, then P (neither A nor B) is equal to

(a) $\frac{2}{3}$
(b) $\frac{1}{6}$
(c) $\frac{5}{6}$
(d) $\frac{1}{3}$

18. Find the values of x, y and z from the following equations $\begin{bmatrix} 4 & x-z \\ 2+y & xz \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -1 & 10 \end{bmatrix}$.]

(a) $x = -5,\ y = 3,\ z = 2$
(b) $x = 5,\ y = -3,\ z = 2$
(c) $x = 5,\ y = 3,\ z = -2$
(d) $x = 5,\ y = -3,\ z = -2$

19. **Assertion:** if $2P(A) = P(B) = \frac{5}{13}$ and $P\left(\frac{A}{B}\right) = \frac{2}{5}$, Then $P(A \cup B)$ is $\frac{11}{26}$

Reason: E_1 and E_2 are two events. then $P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cup E_2)}{P(E_2)}$, $0 < P(E_2) \leq 1$

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.
(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.
(c) Assertion is true; Reason is false.
(d) Assertion is false; Reason is true.

20. Consider, if $u = f(n)$, $v = g(x)$, then the derivative of f with respect to g is $\frac{du}{dv} = \frac{du/dx}{dv/dx}$.

Assertion: Derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is 1 for $0 < x < 1$

Reason: $\sin^{-1}\left(\frac{2x}{1+x^2}\right) \neq \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ for $-1 \leq 1x \leq 1$

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.
(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.
(c) Assertion is true; Reason is false.
(d) Assertion is false; Reason is true.

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SECTION - B

Section B consists of 5 questions of 2 marks each.

21. If a line makes angles 90° and 60°, respectively with the positive directions of X and Y-axes, find the angle which it makes with the positive direction of Z-axis.

22. Evaluate $\int \frac{2}{1+\cos 2x}\,dx$.

OR

Write the value of $\int \frac{x+\cos 6x}{3x^2+\sin 6x}\,dx$.

23. Prove that if E and F are independent events, then the events E and F' are also independent.

24. Find λ and μ, if $(\hat{i}+3\hat{j}+9\hat{k})\times(3\hat{i}-\lambda\hat{j}+\mu\hat{k}) = \vec{0}$.

OR

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then write the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.

25. If $R=\{(x,y): x+2y=8\}$ is a relation on N, then write the range of R.

SECTION - C

Section C consists of 6 questions of 3 marks each.

26. If $y=e^{\tan^{-1}x}$, prove that $(1+x^2)\frac{d^2y}{dx^2}+(2x-1)\frac{dy}{dx}=0$.

27. If $A=\begin{bmatrix}1 & 2\\ 3 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 3\\ -1 & 1\end{bmatrix}$, write the value of $|AB|$.

28. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+\vec{b}$ is also a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

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29. Write the value of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$.

30. Evaluate $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$.

OR

Evaluate $\int \frac{dx}{x(x^5 + 3)}$.

31. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is

(i) strictly increasing

(ii) strictly decreasing.

OR

Find the intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36x}{5} + 11$ is

(i) strictly increasing

(ii) strictly decreasing.

SECTION - D

Section D consists of 4 questions of 5 marks each.

32. A wire of length 28 m is to be cut into two pieces. One of the two pieces is to be made into a square and the other into a circle. What should be the lengths of two pieces, so that the combined area of circle and square is minimum?

OR

Show that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

33. Find $\int \frac{4}{(x-2)(x^2+4)} dx$.

OR

Find $\int \frac{2x}{(x^2+1)(x^2+2)^2} dx$.

Continue on next page......

34. An urn contains 4 white and 6 red balls. Four balls are drawn at random (without replacement) from the urn. Find the probability distribution of the number of white balls?

OR

Find the probability distribution of number of doublets in three tosses of a pair of dice.

35. Find the angle between following pair of lines. $\frac{-x+2}{-2} = \frac{y-1}{7} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z-5}{4}$ and check whether the lines are parallel or perpendicular.

OR

Find the shortest distance between lines whose vector equations are

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$$
$$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}.$$

SECTION - E

Case study based questions are compulsory.

36. Rice is a nutritional staple food which provides instant energy as its most important component is carbohydrate (starch). On the other hand, rice is poor in nitrogenous substances with average composition of these substances being only 8 per cent and fat content or lipids only negligible, i.e., 1 per cent and due to this reason it is considered as a complete food for eating. Rice flour is rich in starch and is used for making various food materials.

Two farmers Ramkishan and Gurcharan Singh cultivate only three varieties of rice namely Basmati, Permal and Naura. The sale (in ₹) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices A and B.

September Sales (in ₹)

$$A = \begin{array}{c} \text{Basmati} \quad \text{Permal} \quad \text{Naura} \\ \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} \begin{array}{l} \text{Ramkishan} \\ \text{Gurcharan Singh} \end{array} \end{array}$$

October Sales (in ₹)

$$B = \begin{array}{c} \text{Basmati} \quad \text{Permal} \quad \text{Naura} \\ \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix} \begin{array}{l} \text{Ramkishan} \\ \text{Gurcharan Singh} \end{array} \end{array}$$

(i) Find the combined sales in September and October for each farmer in each variety.

(ii) Find the decrease in sales from September to October.

(iii) If both farmers receive 2% profit on gross sales, compute the profit for each farmer and for each variety sold in October.

37. Global Air Lines has contracted with a tour group to transport a minimum of 1,600 first-class passengers and 4,800 economy-class passengers from New York to London during a 6-month time period. Global Air has two types of airplanes, the Orville 606 and the Wilbur W-1112. The Orville 606 carries 20 first-class passengers and 80 economy-class passengers and costs $12,000 to operate. The Wilbur W-1112 carries 80 first-class passengers and 120 economy-class passengers and costs $18,000 to operate.

During the time period involved, Global Air can schedule no more than 52 flights on Orville 606s and no more than 30 flights on Wilbur W-1112s.

(i) How should Global Air Lines schedule its flights to minimize its costs?

(ii) What operating costs would this schedule entail?

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38. Chemical reaction, a process in which one or more substances, the reactants, are converted to one or more different substances, the products. Substances are either chemical elements or compounds. A chemical reaction rearranges the constituent atoms of the reactants to create different substances as products.

In a certain chemical reaction, a substance is converted into another substance at a rate proportional to the square of the amount of the first substance present at any time t. Initially $(t = 0)$ 50 g of the first substance was present; 1 hr later, only 10 g of it remained.

(i) Find an expression that gives the amount of the first substance present at any time t.

(ii) What is the amount present after 2 hr?

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Sample Paper 20

Class - 12th Exam - 2024 - 25

Mathematics (Code-041)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided some questions.
9. Use of calculators is not allowed.

SECTION - A

Section A consists of 20 questions of 1 mark each.

1. The value of $\int_{-2}^{2}(x\cos x + \sin x + 1)\,dx$ is

(a) 2 (b) 0

(c) -2 (d) 4

2. If x is measured in degrees, then $\frac{d}{dx}(\cos x)$ is equal to

(a) $-\sin x$ (b) $-\frac{180}{\pi}\sin x$

(c) $-\frac{\pi}{180}\sin x$ (d) $\sin x$

3. The image of the interval $[1, 3]$ under the mapping $f : R \to R$ given by $f(x) = 2x^3 - 24x + 107$ is

(a) $[75, 89]$ (b) $[74, 89]$

(c) $[0, 75]$ (d) none of these

4. $\int_{0}^{\pi/2}\left|\cos\left(\frac{x}{2}\right)\right|dx$ is equal to

(a) 1 (b) -2

(c) $\sqrt{2}$ (d) 0

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5. The area of the region bounded by the lines $y = mx, x = 1,\ x = 2$ and X-axis is 6 sq units, then m is equal to

(a) 3
(b) 1
(c) 2
(d) 4

6. If $A = \{1, 2, 3, 4\}, B = \{1, 2, 3\}$, then number of mappings from A to B is

(a) 3^4
(b) 12
(c) 4^3
(d) 2^7

7. Solution set of the inequality $x \geq 0$ is

(a) Half plane on the left of y-axis
(b) Half plane on the right of y-axis excluding the points of y-axis
(c) Half plane on the right of y-axis including the points on y-axis
(d) None of the above

8. The value of $\sec^2(\tan^{-1}2) + \text{cosec}^2(\cot^{-1}3)$ is

(a) 15
(b) 5
(c) 13
(d) 14

9. If matrix $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ \lambda & -3 & 0 \end{bmatrix}$ is singular, then λ is equal to

(a) -2
(b) -1
(c) 1
(d) 2

10. The order of the differential equation whose general solution is given by $y = (c_1 + c_2)\sin(x + c_3) - c_4 e^{x + c_5}$, is

(a) 5
(b) 4
(c) 2
(d) 3

11. The solution of the differential equation $\frac{dy}{dx} + 1e^{x+y}$

(a) $(x + c)\, e^{x+y} = 0$
(b) $(x + y)\, e^{x+y} = 0$
(c) $(x - c)\, e^{x+y} = 1$
(d) $(x - c)\, e^{x+y} = 0$

12. Solution of $\frac{dy}{dx} + y\sec x = \tan x$ is

(a) $y(\sec x + \tan x) = \sec x + \tan x - x + c$
(b) $y = \sec x + \tan x - x + c$
(c) $y(\sec x + \tan x) = \sec x + \tan x + x + c$
(d) none of the above

13. The vector $\vec{a}$ is equal to

(a) $(\vec{a}\cdot\hat{k})\hat{i}+(\vec{a}\cdot\hat{i})\hat{j}+(\vec{a}\cdot\hat{j})\hat{k}$

(b) $(\vec{a}\cdot\hat{i})\hat{i}+(\vec{a}\cdot\hat{j})\hat{j}+(\vec{a}\cdot\hat{k})\hat{k}$

(c) $(\vec{a}\cdot\hat{j})\hat{i}+(\vec{a}\cdot\hat{k})\hat{j}+(\vec{a}\cdot\hat{i})\hat{k}$

(d) $(\vec{a}\cdot\vec{a})(\hat{i}+\hat{j}+\hat{k})$

14. Let $\vec{a},\vec{b}$ and $\vec{c}$ be vectors of magnitude 3, 4 and 5 respectively. If $\vec{a}$ is perpendicular to $(\vec{b}+\vec{c}),\vec{b}$ is perpendicular to $(\vec{c}+\vec{a})$ and $\vec{c}$ is perpendicular to$(\vec{a}+\vec{b})$, then the magnitude of the vector $\vec{a}+\vec{b}+\vec{c}$ is

(a) 5

(b) $5\sqrt{2}$

(c) $5\sqrt{3}$

(d) 2

15. The least, value of the function $f(x) = ax + b/x$, $a > 0$, $b > 0$, $x > 0$ is

(a) $\sqrt{ab}$

(b) $2\sqrt{\frac{a}{b}}$

(c) $2\sqrt{\frac{b}{a}}$

(d) $2\sqrt{ab}$

16. A bag A contains 4 green and 3 red balls and bog B contains 4 red and 3 green balls. One bag is taken at random and a ball is drawn and noted to be green. The probability that it comes from bag B is

(a) $\frac{2}{7}$

(b) $\frac{2}{3}$

(c) $\frac{3}{7}$

(d) $\frac{1}{3}$

17. The equation of the line through the point $(2,3,-5)$ and equally inclined to the axes are

(a) $x-2=y-3=z+5$

(b) $\frac{x-1}{2}=\frac{y-1}{3}=\frac{z-1}{-5}$

(c) $\frac{x}{2}=\frac{y}{3}=\frac{z}{-5}$

(d) none of these

18. Let $f(x)=x|x|$, then $f(x)$ has a

(a) local maxima at $x=0$

(b) local minima at $x=0$

(c) point of inflexion at $x=0$

(d) none of the above

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19. Consider the function $f(x) = \begin{cases} x^2, & x \geq 1 \\ x+1, & x < 1 \end{cases}$

Assertion: f is not derivable at $x = 1$ as $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$

Reason: If a function f is derivable at a point a then it is continuous at a

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

20. **Assertion:** Scalar matrix $A = [a_{ij}] = \begin{cases} k: & i = j \\ 0: & i \neq j \end{cases}$ where k is a scalar, in an identity matrix when $k = 1$

Reason: Every identity matrix is not a scalar matrix.

(a) Assertion is true, Reason is true; Reason is a correct explanation for Assertion.

(b) Assertion is true, Reason is true; Reason is not a correct explanation for Assertion.

(c) Assertion is true; Reason is false.

(d) Assertion is false; Reason is true.

SECTION - B

Section B consists of 5 questions of 2 marks each.

21. If a line has direction ratios $(2, -1, -2)$, then what are its direction cosines?

22. Let R is the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$. Write the equivalence class $[0]$.

23. Write the projection of $(\vec{b} + \vec{c})$ on $\vec{a}$, where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.

OR

If $\vec{a}$ and $\vec{b}$ are to vectors such that $|\vec{a} + \vec{b}| = |\vec{a}|$,the prove that vector $2\vec{a} + \vec{b}$ is perpendicular to vector $\vec{b}$.

24. Two dice are thrown n times in succession. What is the probability of obtaining a doublet six at-least once?

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25. Evaluate $\int \frac{(\log x)^2}{x} dx$.

OR

Evaluate $\int \frac{e^{\tan^{-1}x}}{1+x^2} dx$.

SECTION - C

Section C consists of 6 questions of 3 marks each.

26. If $x = \cos t(3 - 2\cos^2 t)$ and $y = \sin t(3 - 2\sin^2 t)$, then find the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

27. If $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 5$, $|\vec{b}| = 6$ and $|\vec{c}| = 9$, then find the angle between $\vec{a}$ and $\vec{b}$.

28. Solve the following equation for x.

$$\cos(\tan^{-1}x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$$

29. Find $|AB|$, if $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$.

30. Find the intervals in which the function given by $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is
(i) increasing
(ii) decreasing.

OR

Show that of all the rectangles of given area, the square has the smallest perimeter.

31. Evaluate $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx$.

OR

Find $\int \frac{x^3}{x^4 + 3x^2 + 2} dx$.

Continue on next page......

SECTION - D

Section D consists of 4 questions of 5 marks each.

32. Evaluate $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$.

33. Maximise $Z = 5x + 3y$ subject to the constraints: $3x + 5y \leq 15$; $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

OR

Minimize $Z = 3x + 5y$ such that $x + 3y \geq 3$, $x + y \geq 2$, x, $y \geq 0$.

34. Find the particular solution of the differential equation $x\frac{dy}{dx} - y + x \operatorname{cosec}\left(\frac{y}{x}\right) = 0$

OR

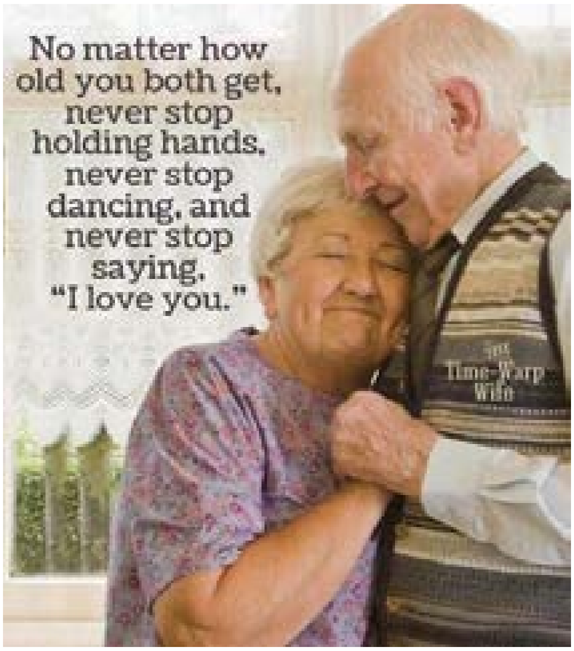
Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, given that $y = 0$ when $x = 1$.

35. Find the vector and Cartesian equations of the line passing through the point $(2,1,3)$ and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$.

SECTION - E

Case study based questions are compulsory.

36. Measures of joint and survivor life expectancy are potentially useful to those designing or evaluating policies affecting older couples and to couples making retirement, savings, and long-term care decisions. However, couple-based measure of life expectancies are virtually unknown in the social sciences.



The odds against a husband who is 45 yr old, living till he is 70 are. 7 : 5 and the odds against his wife who is now 36, living till she is 61 are 5 : 3.
On the basis of above information, answer the following questions.

(i) Find the probabilities of husband living till 70 and wife living till 61.

(ii) Find the probability P(couple will be alive 25 yr hence).

(iii) Find the probability P(exactly one of them will be alive 25 yr hence).

OR

Find the probability P(none of them will be alive 25 yr hence) and probability P(atleast one of them will be alive 25 yr hence).

37. RK Verma is production analysts of a ready-made garment company. He has to maximize the profit of company using data available. He find that $P(x) = -6x^2 + 120x + 25000$ (in Rupee) is the total profit function of a company where x denotes the production of the company.

Based on the above information, answer the following questions.

(i) Find the profit of the company, when the production is 3 units.

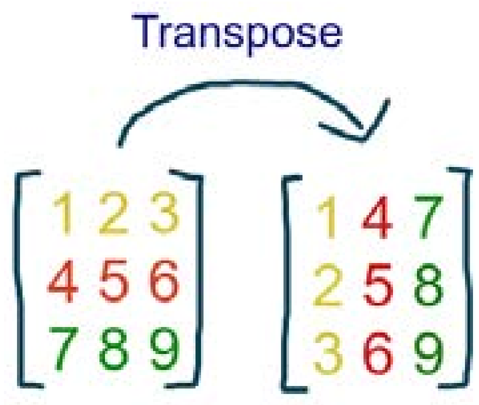
(ii) Find P' (5)

(iii) Find the interval in which the profit is strictly increasing.

OR

Find the production, when the profit is maximum.

38. In linear algebra, the transpose of a matrix is an operator which flips a matrix over its diagonal; that is, it switches the row and column indices of the matrix A by producing another matrix, often denoted by A^T. The transpose of a matrix was introduced in 1858 by the British mathematician Arthur Cayley.



If $A = [a_{ij}]$ be a $m \times n$ matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A. A square matrix $A = [a_{ij}]$ is said to be symmetric, $A^T = A$ for all possible values of i and j. A square matrix A $[a_{ij}]$ is said to be skew-symmetric, if $A^T = -A$ for all possible values of i and j. Based on the above, information, answer the following questions.

(i) Find the transpose of [1, -2, -5].

(ii) Find the transpose of matrix (ABC).

(iii) Evaluate $(A+B)^T - A,. A = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

OR

Evaluate $(AB)^T$, where $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

□□□□□□□